



UNIVERSITÄT LEIPZIG

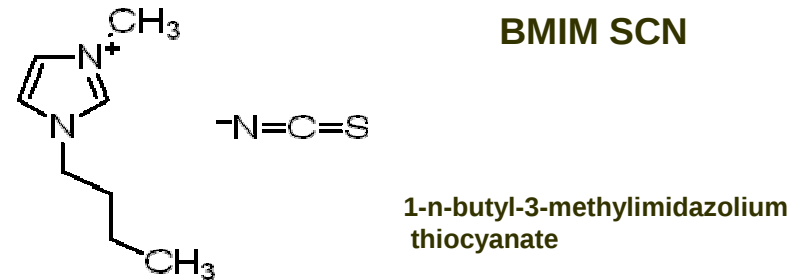
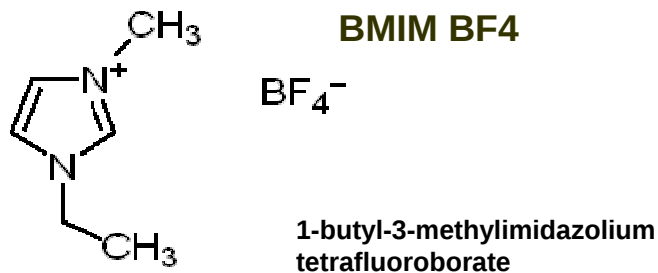
The dielectric properties of glassy ion-conducting materials

F.Kremer

Co-authors: J.R. Sangoro, A.Serghei,
C. Iacob, S. Naumov, J. Kärger

Example for ion-conducting glassy materials: Ionic Liquids (ILs)

Ionic liquids are **salts between an organic cation and a „simple“ anion:**



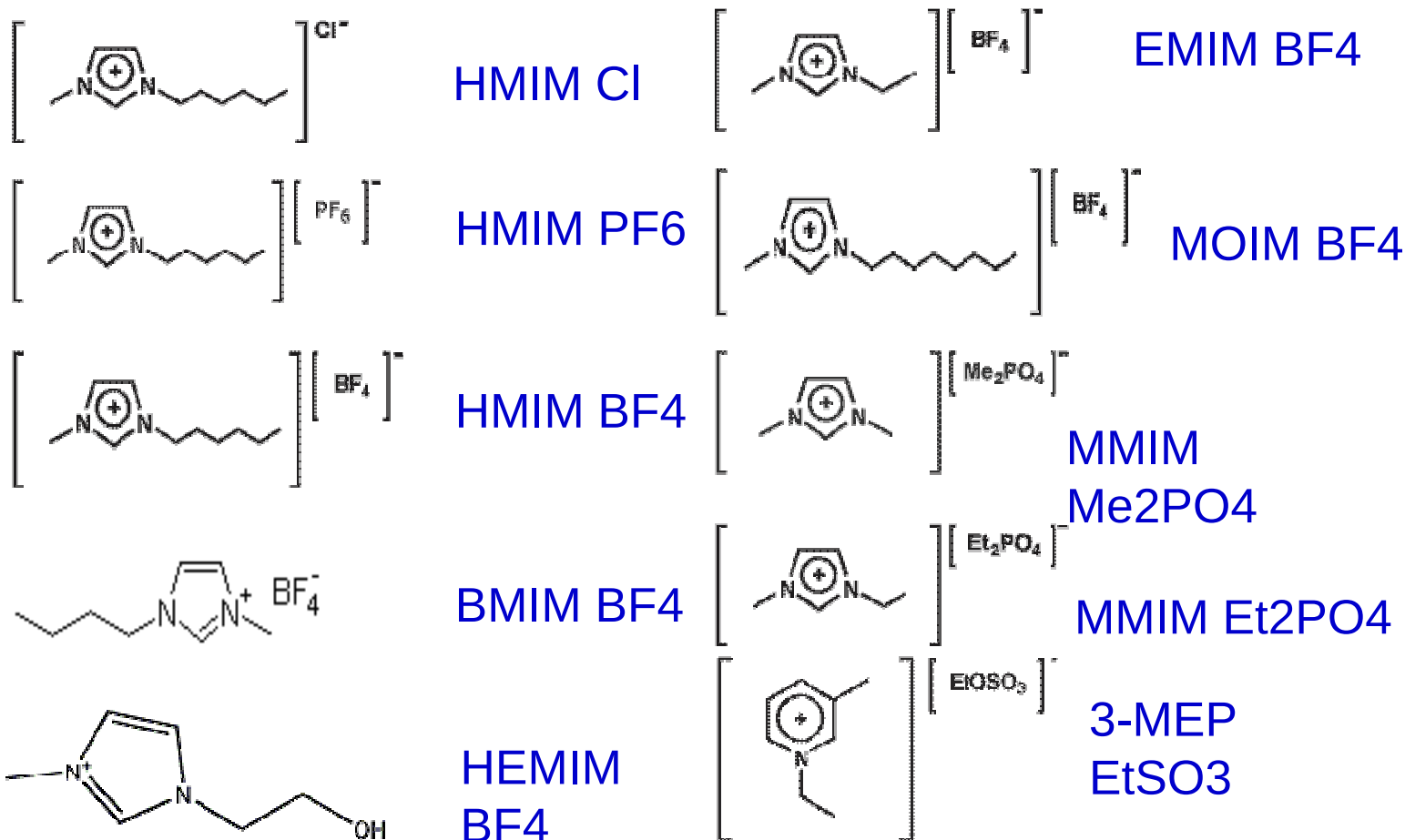
Properties of ILs:

- Low glass transition temperature (220 K)
- High conductivity at room temperature
- Low viscosity even at low temperature (200 K)
- Novel solvent

Applications of ILs:

- Medium in fuel cells, super-capacitors, batteries and solar cells
- Catalysis, extraction and separation processes
- Lubricants and fuels

Structures of some of the ILs studied:



Basic relations between the complex dielectric function ϵ^* and the complex conductivity σ^*

The *linear* interaction of electromagnetic fields with matter is described by **Maxwell's equations**

$$\text{curl } H = j + \frac{\partial D}{\partial t}$$

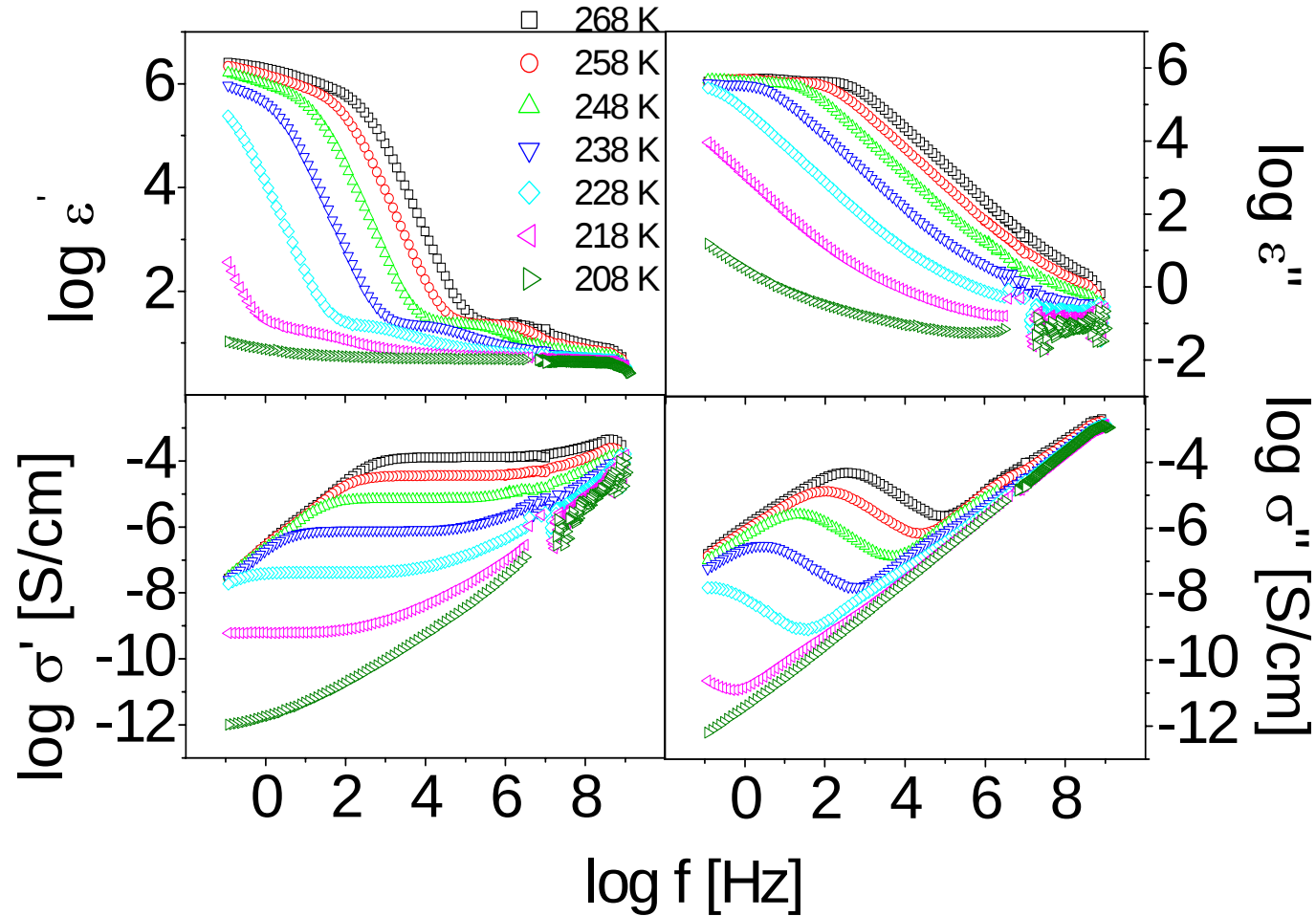
$$D = \epsilon^* \epsilon_0 E \qquad j = \sigma^* E \quad (\text{Ohm's law})$$

(Current-density and the time derivative of D are equivalent)

$$\epsilon^*(\omega) = \epsilon' - i\epsilon'' \qquad \sigma^*(\omega) = \sigma' + i\sigma''$$

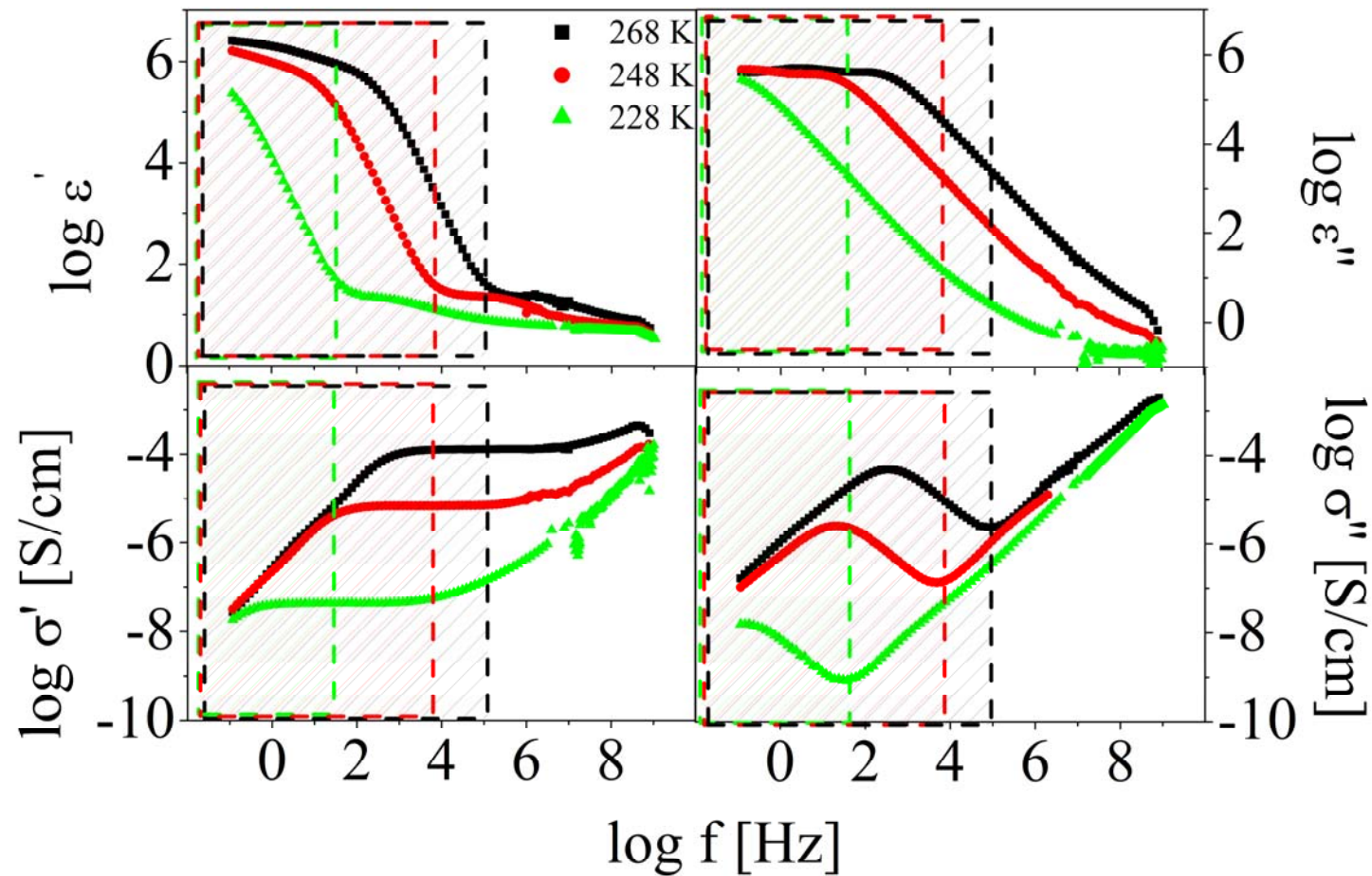
$$\sigma^* = i\omega\epsilon_0\epsilon^*$$

Broadband dielectric spectra of an ionic liquid (MMIM) Me₂PO₄



**Strong temperature dependence of the charge transport processes
and electrode polarisation**

Two temperature dependent regimes: Electrode polarisation and bulk charge transport



The spectral range where electrode polarisation dominates depends strongly on temperature

Basic relations between rotational and translational diffusion

Stokes-Einstein relation:

$$D = kT / \zeta$$

D: diffusion coefficient, $\zeta (=6\pi \eta a)$:
frictional coefficient, T: temperature, k:
Boltzmann constant, a: radius of molecule

Maxwell's relation:

$$\eta = \tau_{\eta} G^{\infty}$$

G^{∞} : instantaneous shear modulus ($\sim 10^8 - 10^{10}$ Pa)

η : viscosity, τ_{η} : structural relaxation time

Einstein-Smoluchowski relation:

$$D = \frac{\lambda^2 \omega_c}{2}$$

λ : characteristic (diffusion) length

ω_c : characteristic (diffusion) rate

Basic electrodynamics and Einstein relation:

$$\sigma_0 = q\mu n = n \frac{q^2 D}{kT} \Rightarrow \sigma_0 \propto \omega_c$$

σ_0 : : dc conductivity, μ : mobility ; q:
elementary charge, n: effective number
density of charge carriers

Predictions **to be** checked experimentally:

1.: $\omega_c = P \omega_\eta$

ω_c : characteristic (diffusion or hopping)
rate

2.: $P = \frac{kT}{2\pi^2 G_\infty \lambda^2 a_s}$

ω_η : structural relaxation rate

G_∞ : instantaneous shear modulus ($\sim 10^8$ Pa
for ILs),

a_s : Stokes' hydrodynamic radius $\sim \lambda$,

k : Boltzmann constant,

λ : characteristic diffusion length $\sim .2$ nm

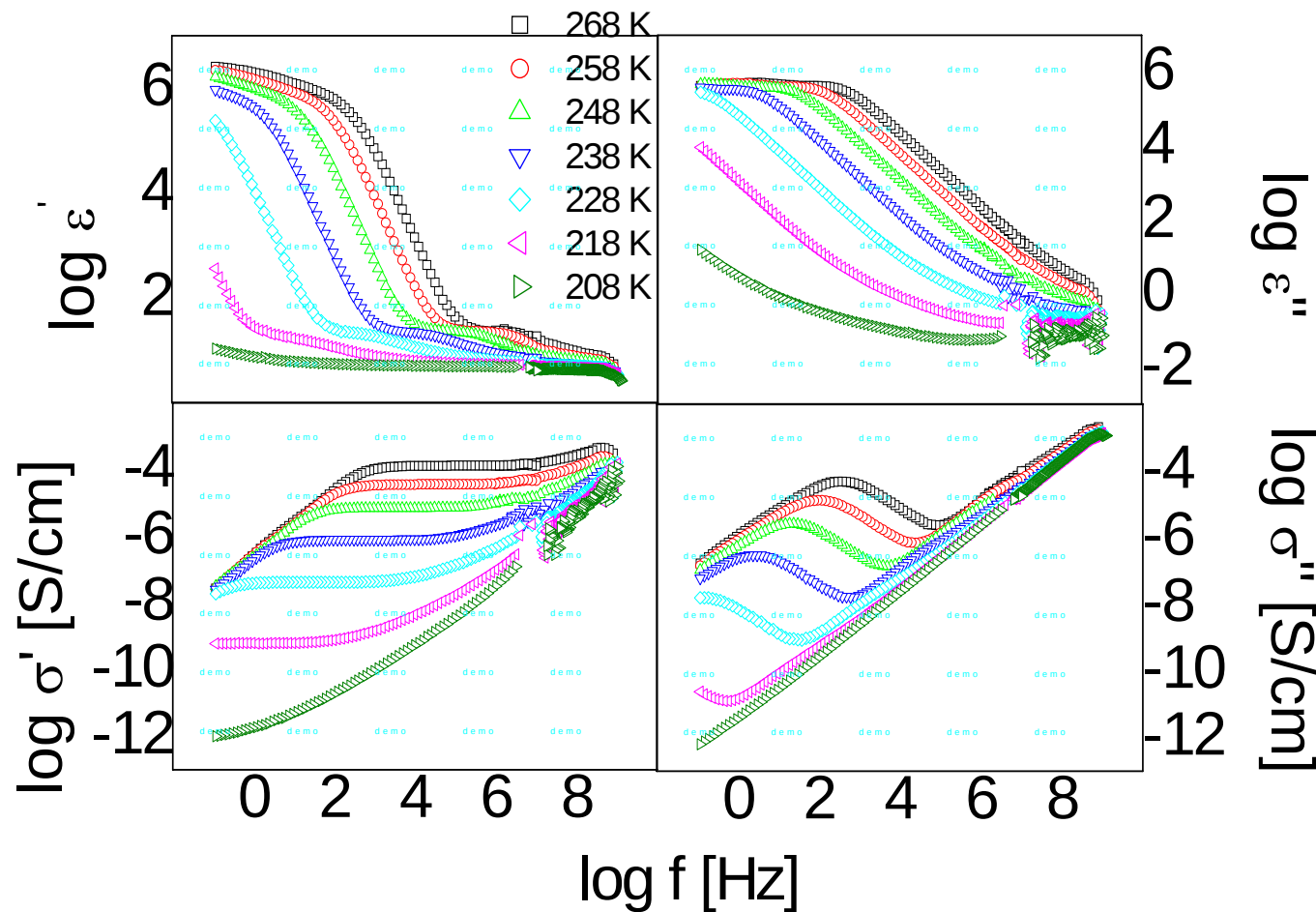
D : molecular diffusion coefficient

4.: $\sigma_0 \sim \omega_c$

(Barton-Nakajima-Namikawa (BNN) relation)

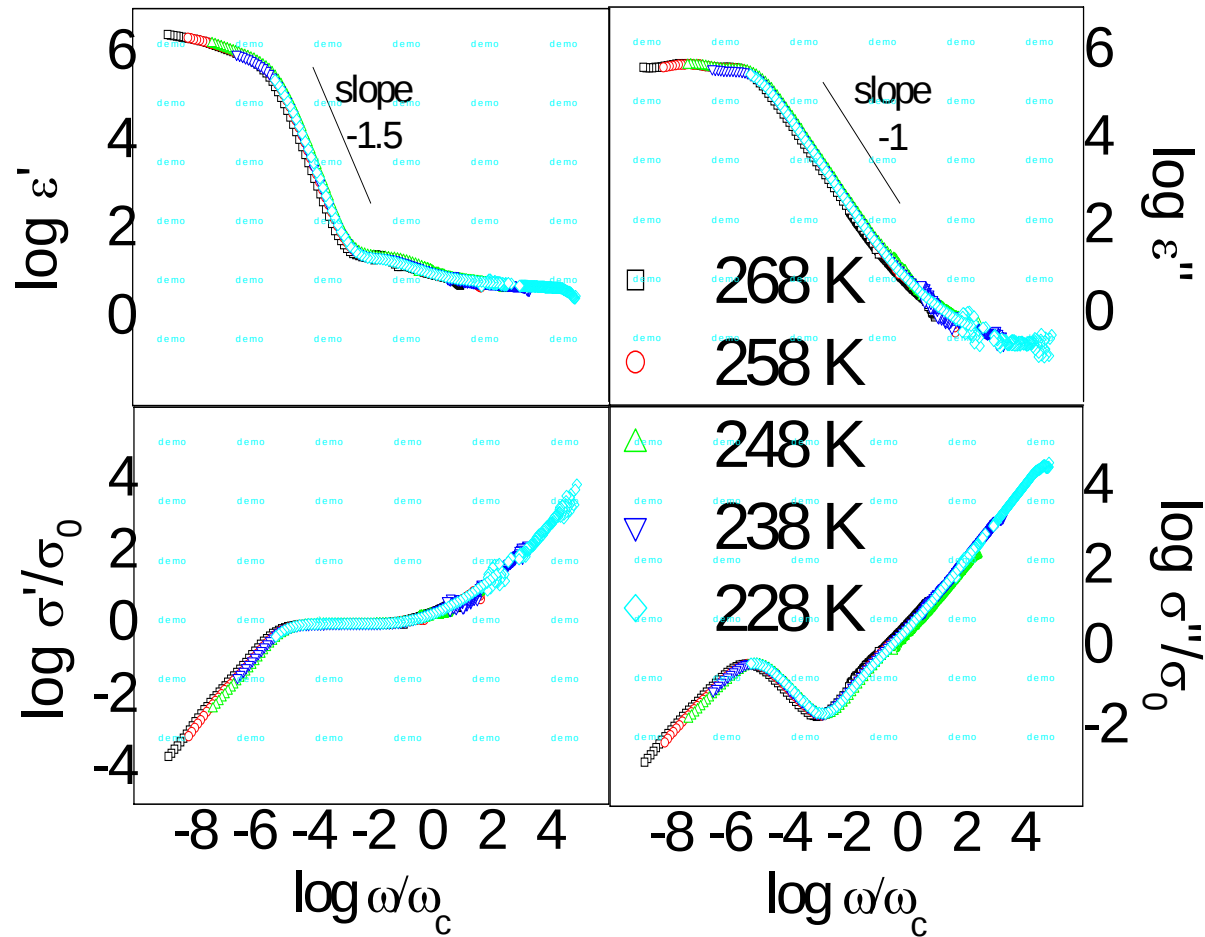
Measurement techniques required: Broadband Dielectric Spectroscopy (BDS);
Pulsed Field Gradient (PFG)-NMR; viscosity measurements;

Dielectric spectra of MMIM Me₂PO₄ ionic liquid



Strong temperature dependence of the charge transport processes and electrode polarisation

Scaling of dielectric spectra with characteristic rate ω_c



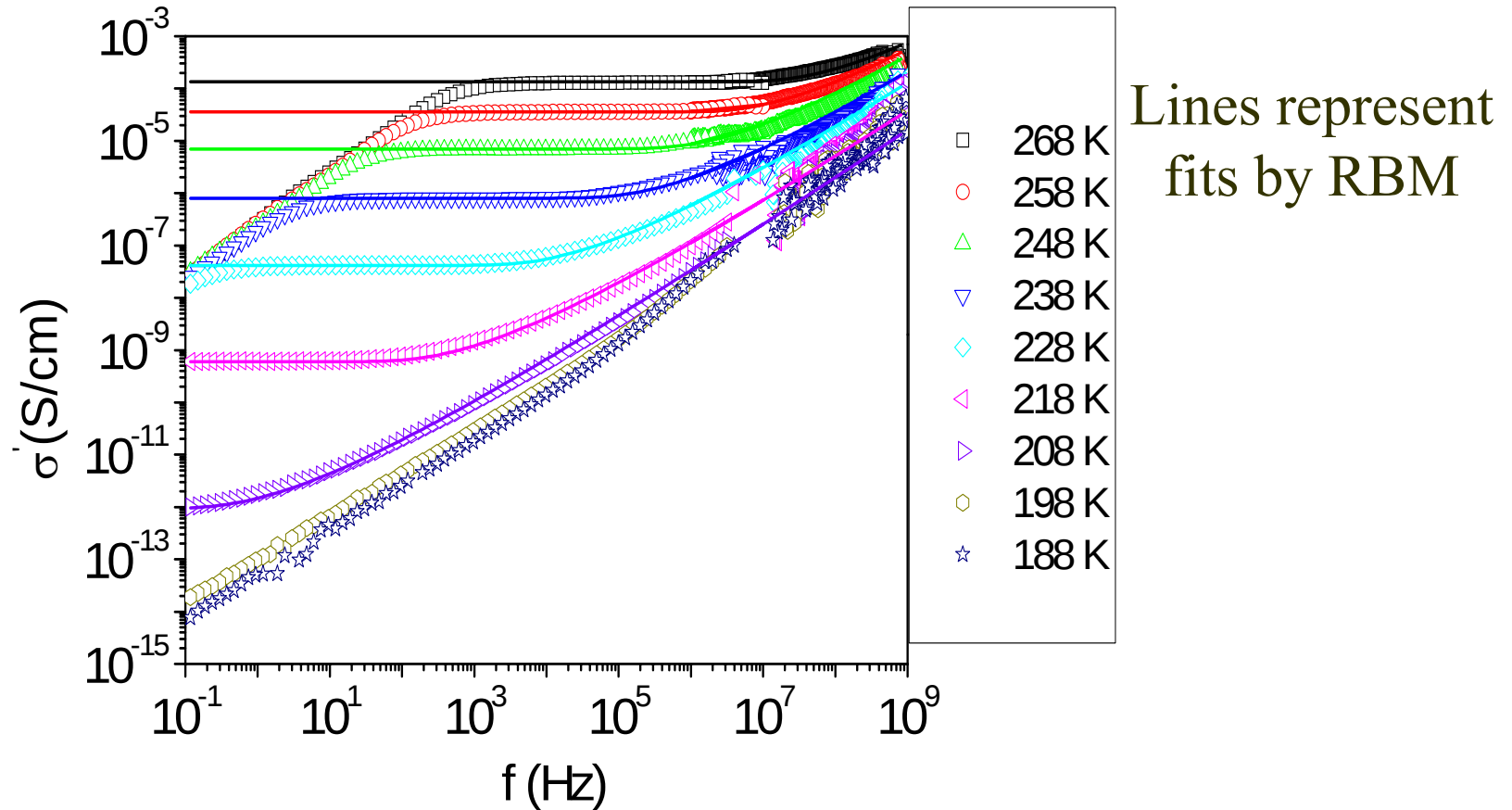
All charts collapse into one if scaled with respect to ω_c . **Identical thermal activation** of charge transport and electrode polarisation

Description of the data: Random Barrier Model (Jeppe Dyre et al.)

- Hopping conduction in a **spatially randomly varying energy landscape**
- Analytic solution obtained within Continuous-Time-Random Walk (CTRW) approximation
- The largest energy barrier determines Dc conduction
- The complex conductivity is described by:

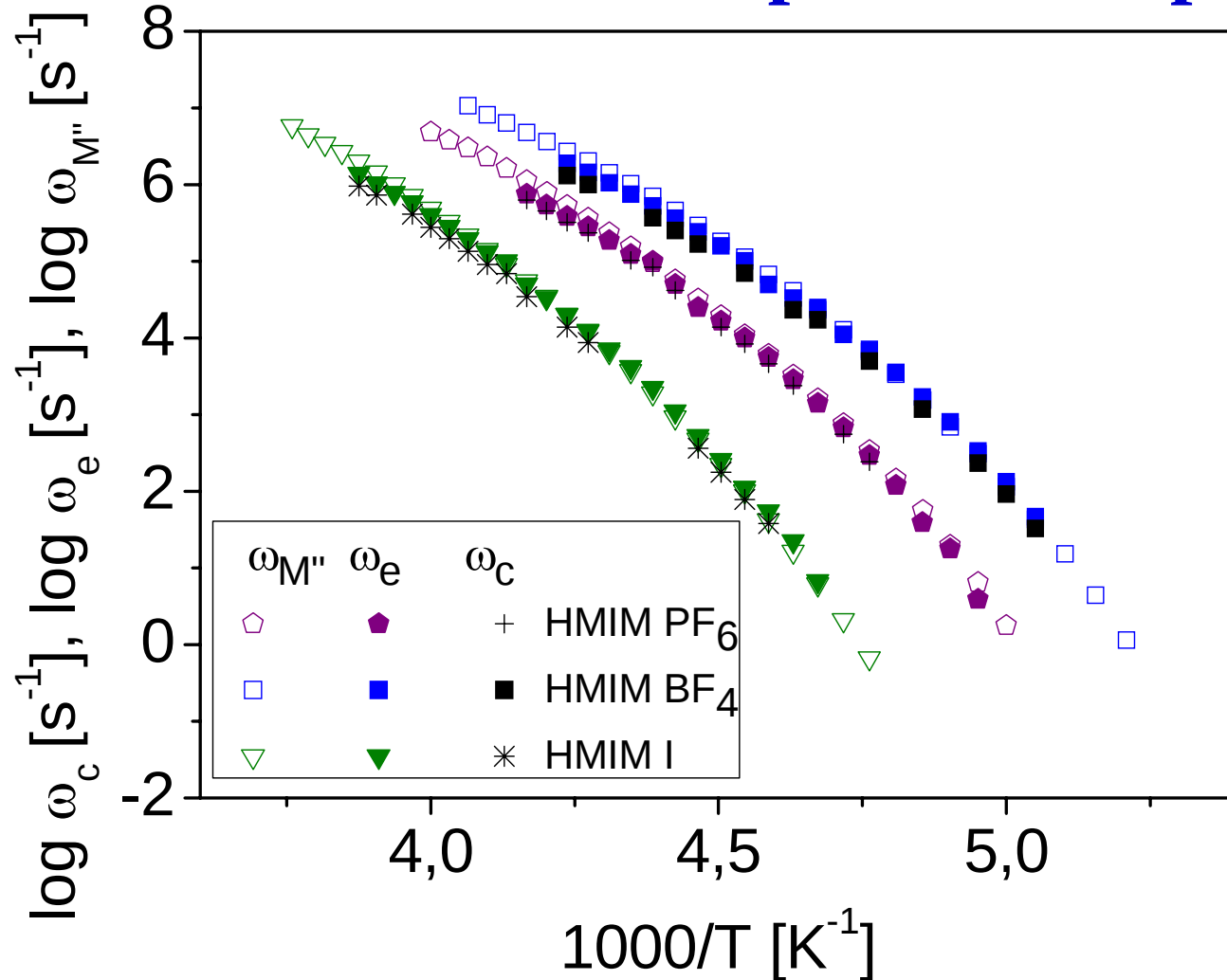
$$\sigma(\omega) = \sigma_0 \left(\frac{i\omega\tau_e}{\ln(1+i\omega\tau_e)} \right) \quad \tau_e \text{ is the characteristic time related to attempt frequency to overcome the largest barrier determining the Dc conductivity}$$

Random Barrier Model (RBM) used to fit the conductivity spectra of ionic liquid (MMIM Me2PO4)



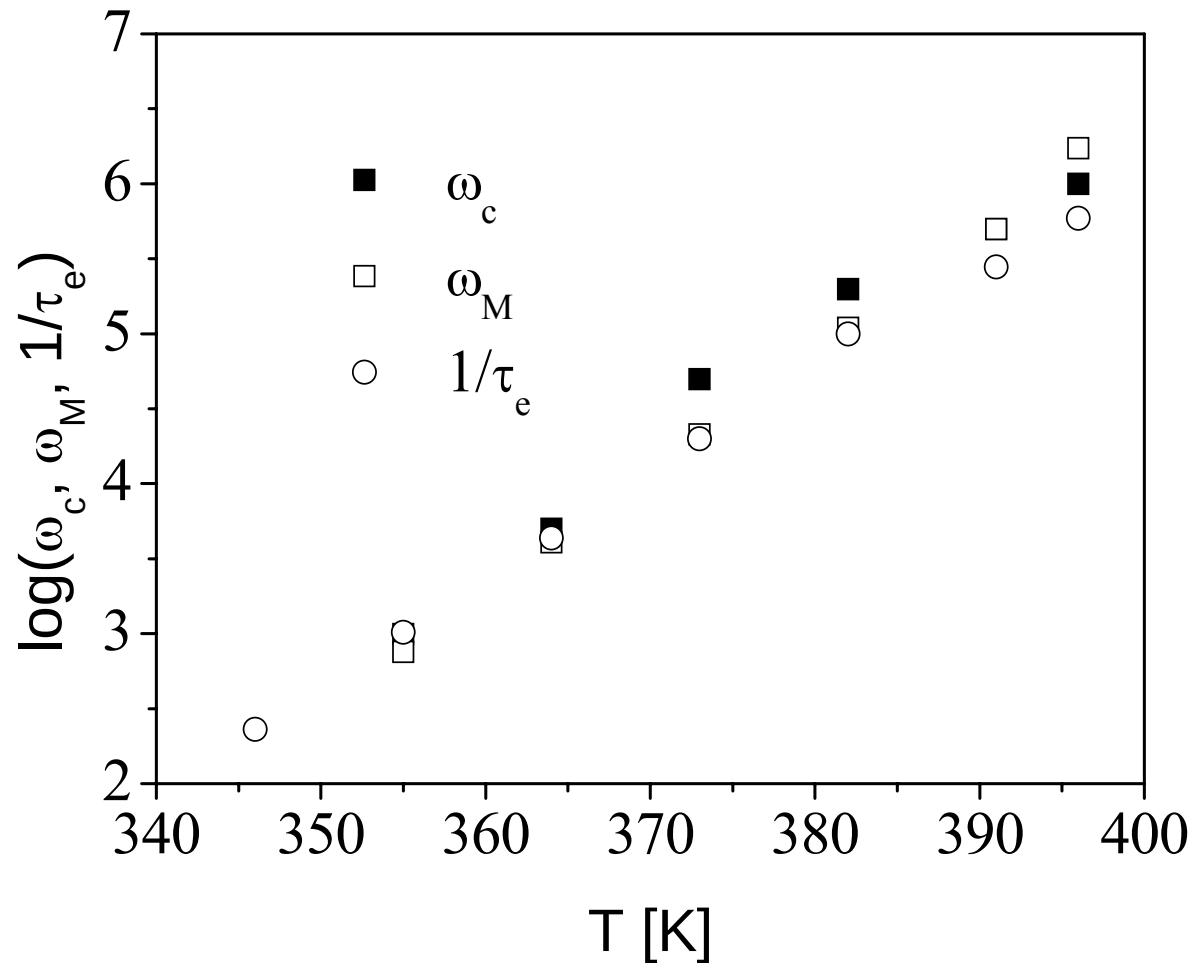
The random barrier model **quantitatively** describes the measured dielectric spectra describing charge transport

The rates ω_c , ω_M and $1/\tau_e$ coincide and have - over 6 decades - a similar temperature dependence



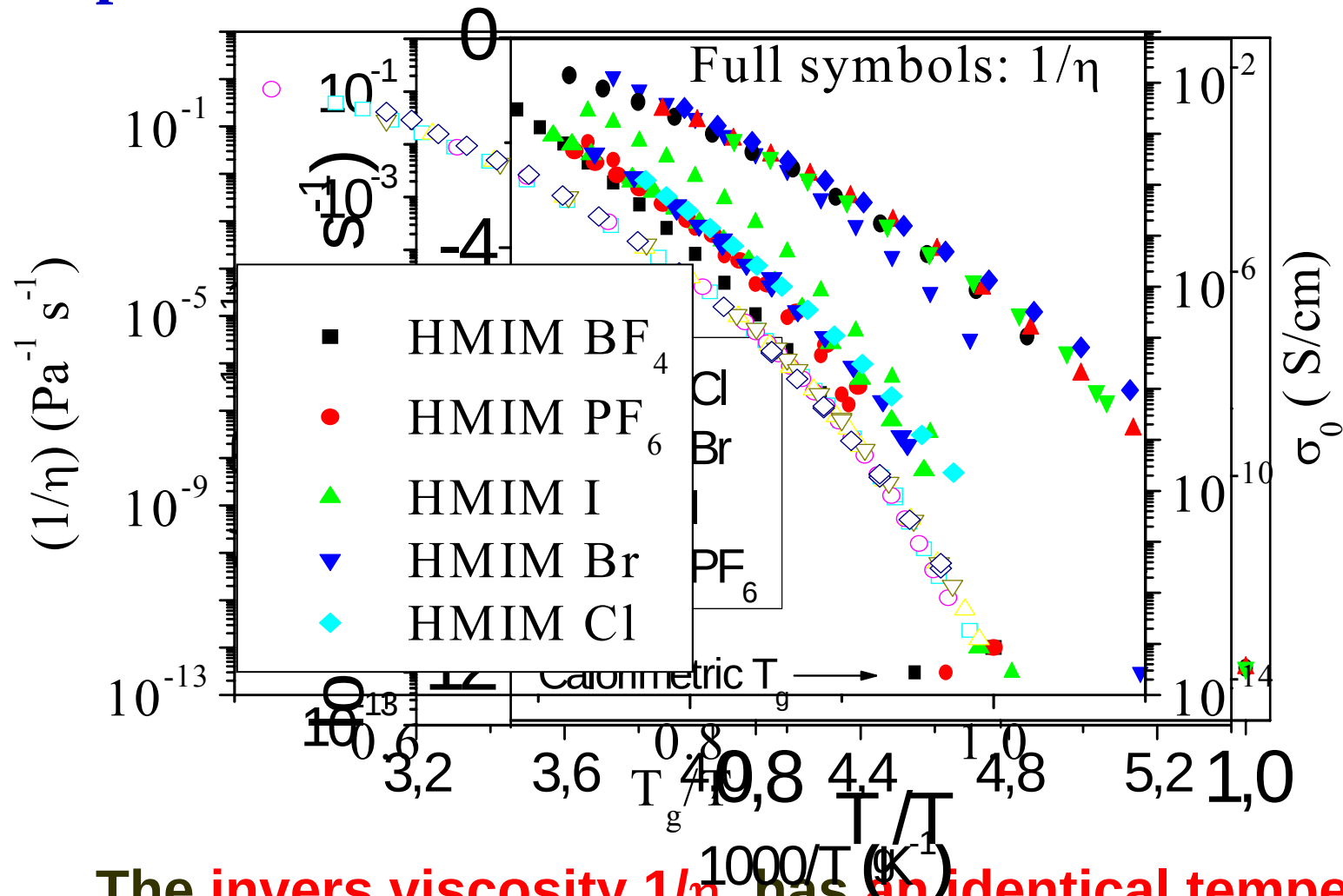
The different rates ω_c , ω_M and $1/\tau_e$ describe the same mechanism of charge transport

The rates ω_c , ω_M and $1/\tau_e$ nearly coincide and have - over 5 decades - a similar temperature dependence



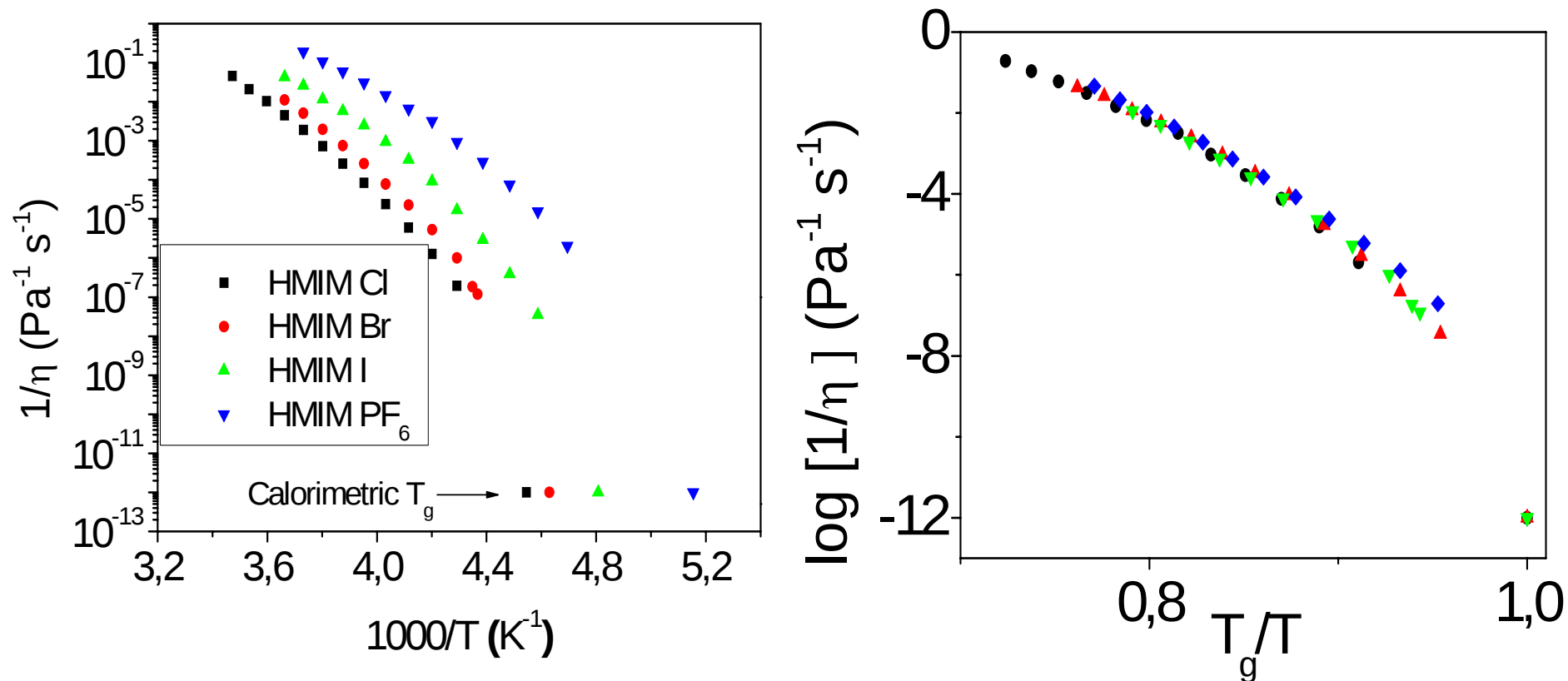
F. Kremer and A. Schönhal's (2003), *Broadband Dielectric Spectroscopy*

Scaling of invers viscosity $1/\eta$ and conductivity σ_0 with temperature



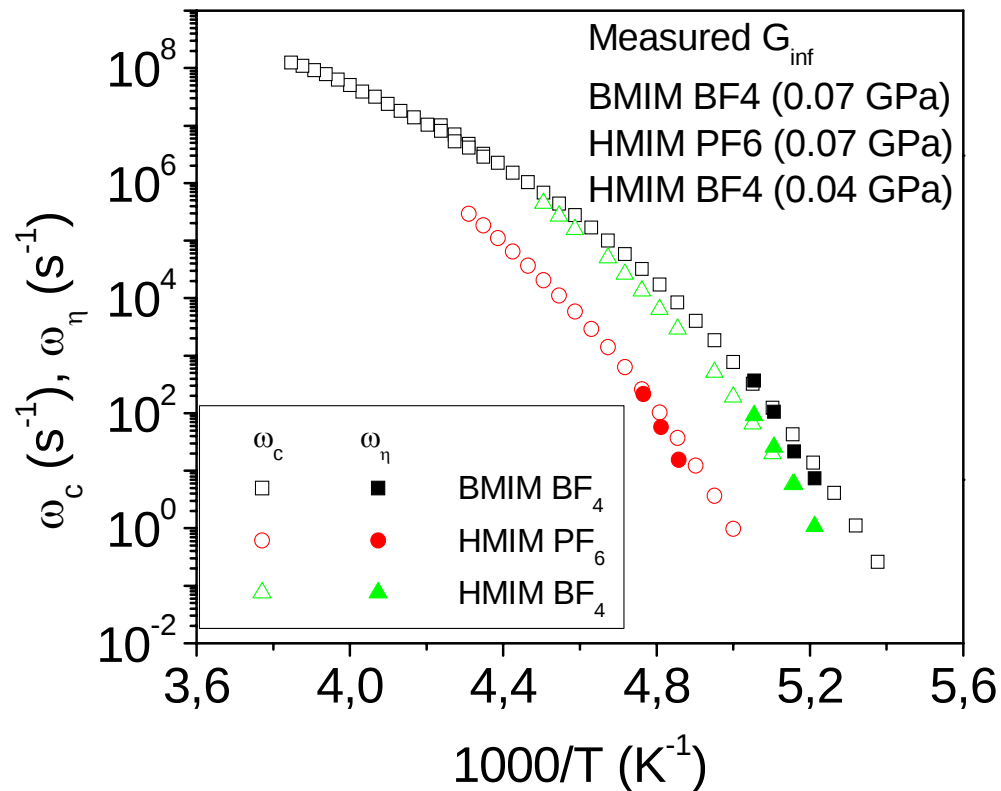
The invers viscosity $1/\eta$ has an identical temperature dependence as σ_0 and scales with T_g .

Effect of structural variation on viscosity



Fluidity ($1/\eta$) strongly depends on the anion ($\text{PF}_6 < \text{I} < \text{Br} < \text{Cl}$)

Correlation between translational and rotational diffusion



Stokes-Einstein, Einstein-Smoluchowski and Maxwell relations:

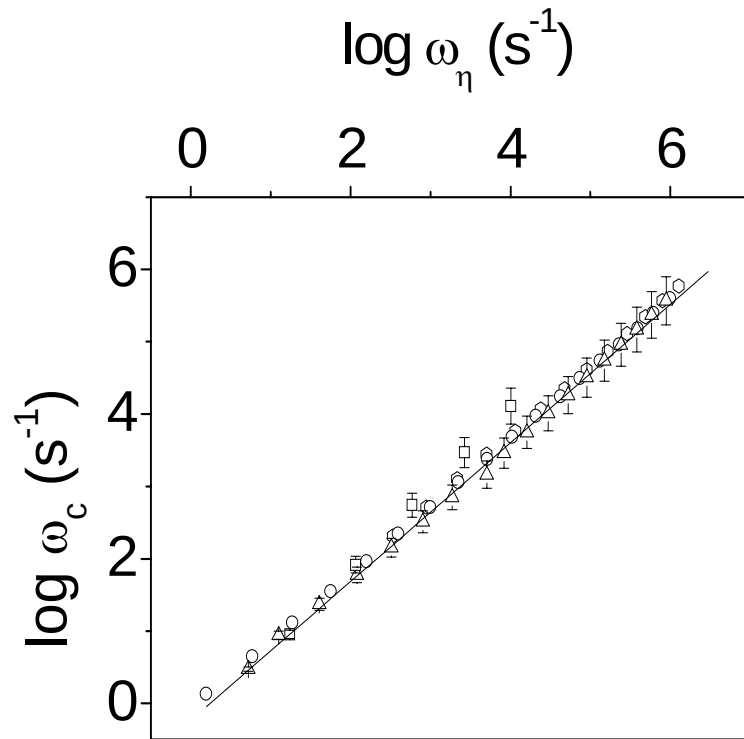
$$\omega_c = P \omega_\eta$$

$$P = \frac{kT}{2\pi^2 G_\infty \lambda^2 a_s} \quad \square \quad 1$$

Typically: $G_\infty \cong 0.1 \text{ GPa}$; $\lambda \cong .2 \text{ nm}$; $a_s \cong .1 \text{ nm}$;

J. R. Sangoro et al. Phys. Chem. Chem. Phys, DOI: 0.1039/b816106b,(2009) .

Correlation between translational and rotational diffusion



Stokes-Einstein, Einstein-Smoluchowski and Maxwell relations:

$$\omega_c = P \omega_{\eta}$$

$$P = \frac{k T}{2 \pi^2 G_{\infty} \lambda^2 a_s} \quad \square \quad 1$$

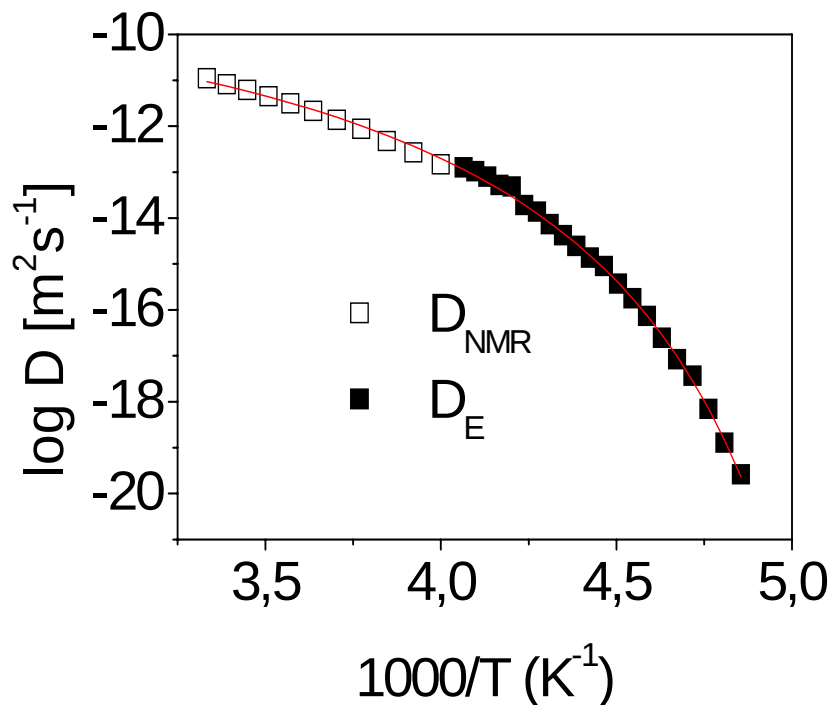
Typically: $G_{\infty} \cong 0.1 \text{ GPa}$; $\lambda \cong .2 \text{ nm}$; $a_s \cong .1 \text{ nm}$;

J. R. Sangoro et al. Phys. Chem. Chem. Phys, DOI: 0.1039/b816106b,(2009) .

Comparison with Pulsed-Field-Gradient NMR and determination of diffusion coefficients from dielectric spectra

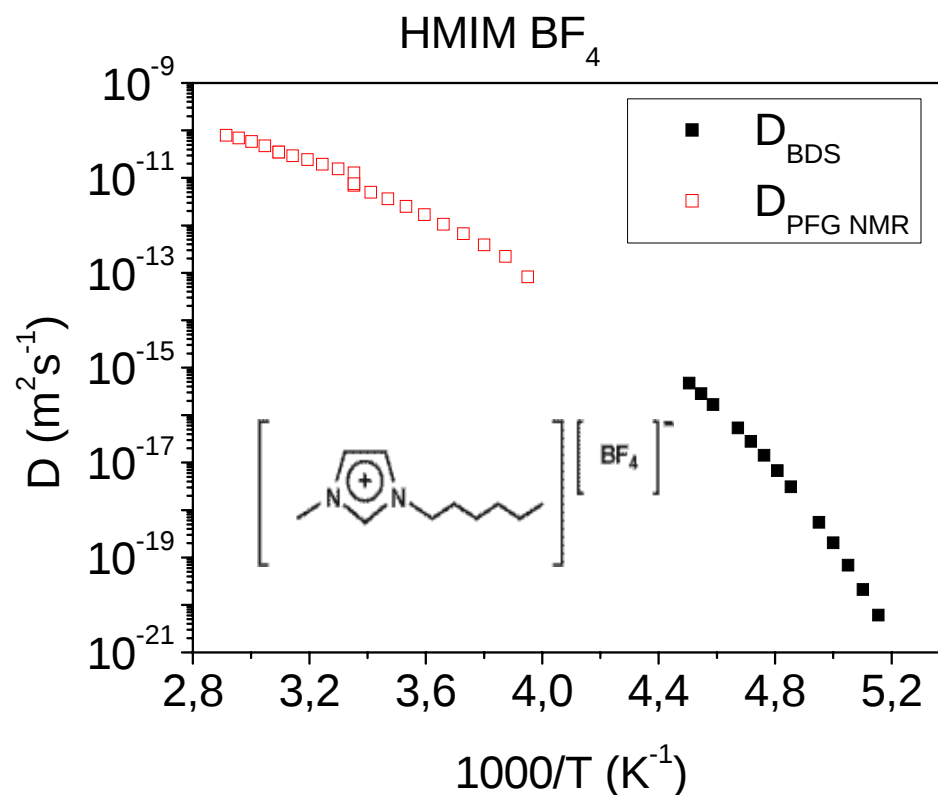
From Einstein and Einstein-Smoluchowski relations and **assuming** the root mean square diffusion distance similar to the order of Pauling diameters, i.e. $\lambda \sim 0.2$ nm follows (**idea: Joshua Rume Sangoro**):

$$\sigma_0(T) = n(T)q\mu(T) = n(T)q \frac{qD(T)}{kT} = \frac{n(T)q^2}{kT} \frac{\lambda^2}{2\tau_h(T)} \sim \omega_e$$



1. **Quantitative agreement** between PFG-NMR measurements and the dielectric determination of the diffusion coefficients.
2. In ILs **mass diffusion** (as measured by PFG-NMR) **equals charge transport** as measured by BDS but in a **much wider spectral range**.
3. The empirical **BNN-relation** is an **immediate consequence**.

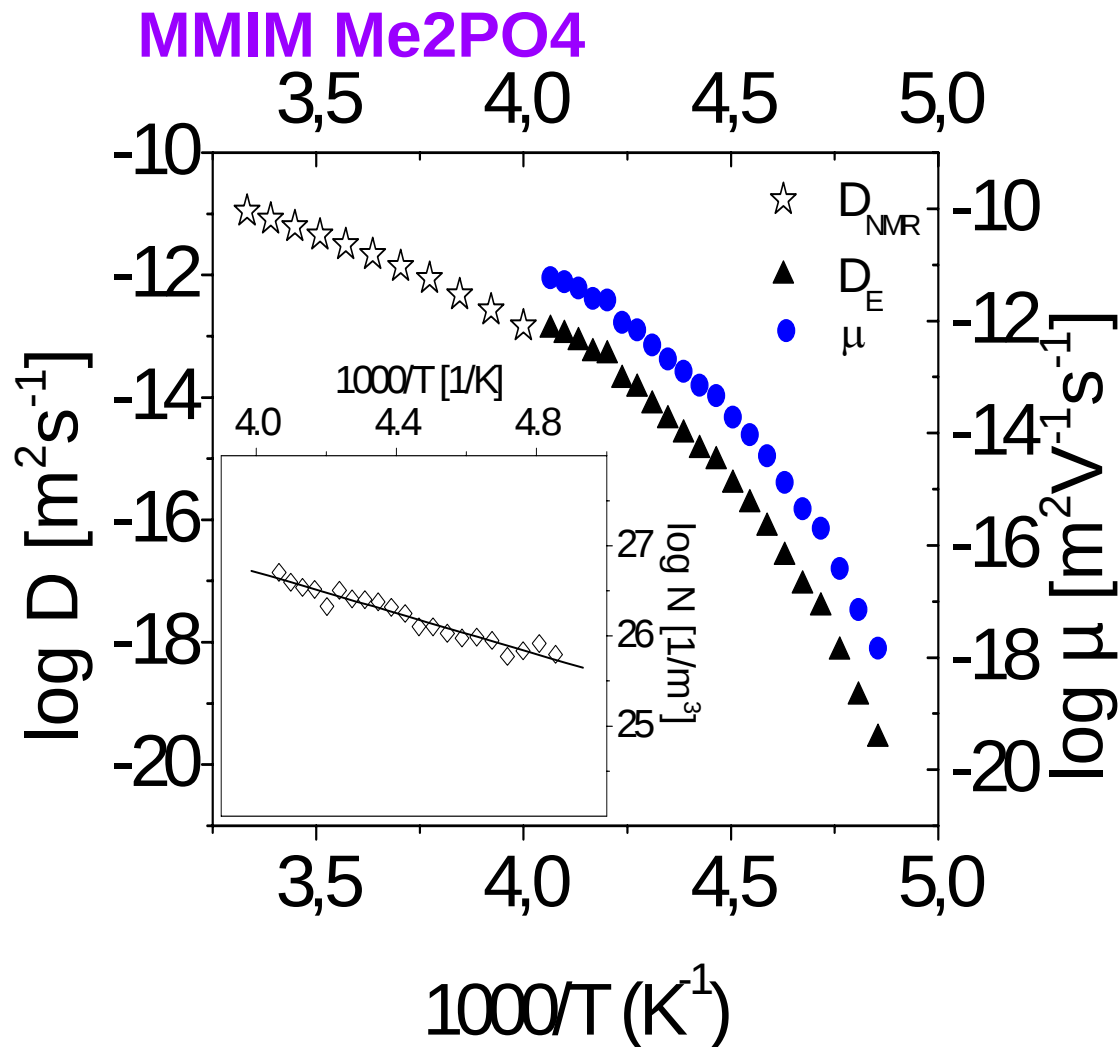
Further diffusion coefficients from dielectric spectra



Charge transport is mass transport in ionic liquids. Probing Fluoride and Hydrogen yield practically the same diffusion coefficient!

PFG NMR measurements performed by Sergej Naumov

Separation of $n(T)$ and $\mu(T)$ from $\sigma_0(T)$



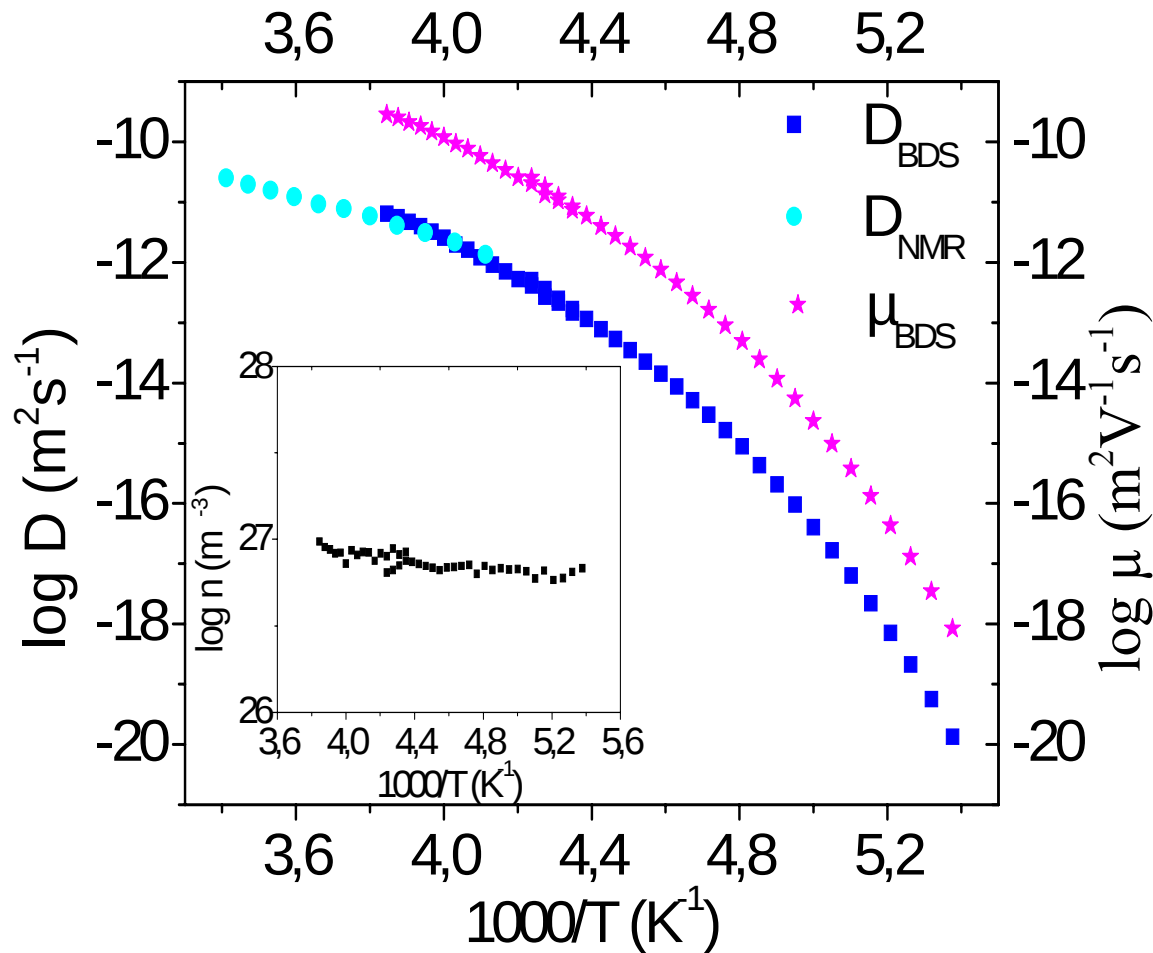
1. $\mu(T)$ shows a VFT-type temperature dependence

2. $n(T)$ shows an Arrhenius-type temperature dependence

3. $\sigma_0(T)$ derives its dependence from $\mu(T)$

Separation technique applied to BMIM BF₄

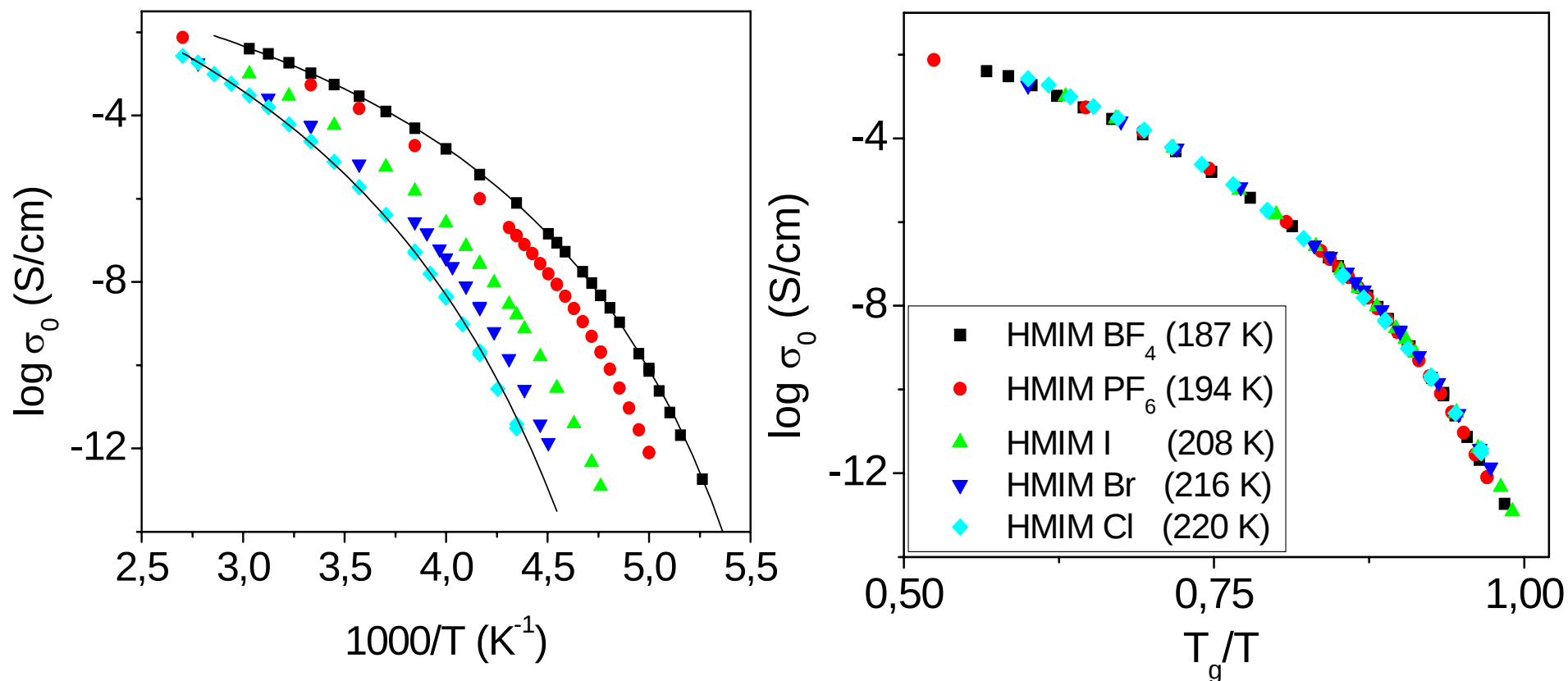
BMIM BF₄



Agreement between dielectric and PFG NMR diffusion coefficients

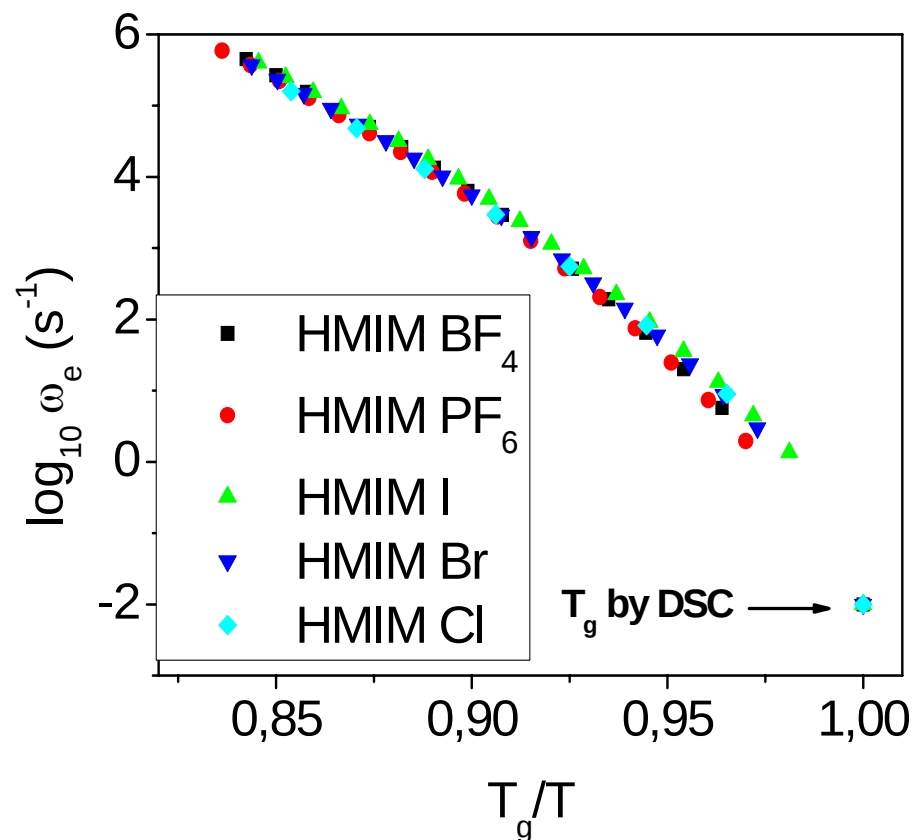
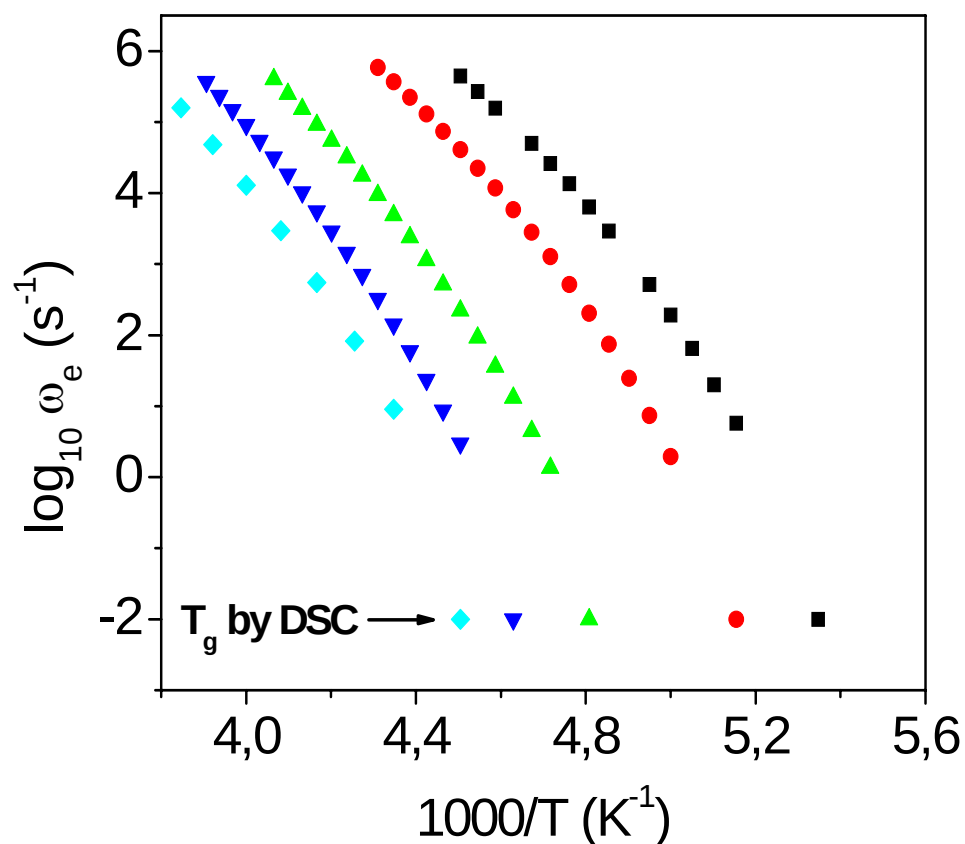
J. R. Sangoro,...S. Naumov, J. Kärger,..., J. Chem. Phys. 128, 212509 (2008)

Temperature dependence of Dc conductivity of ILs



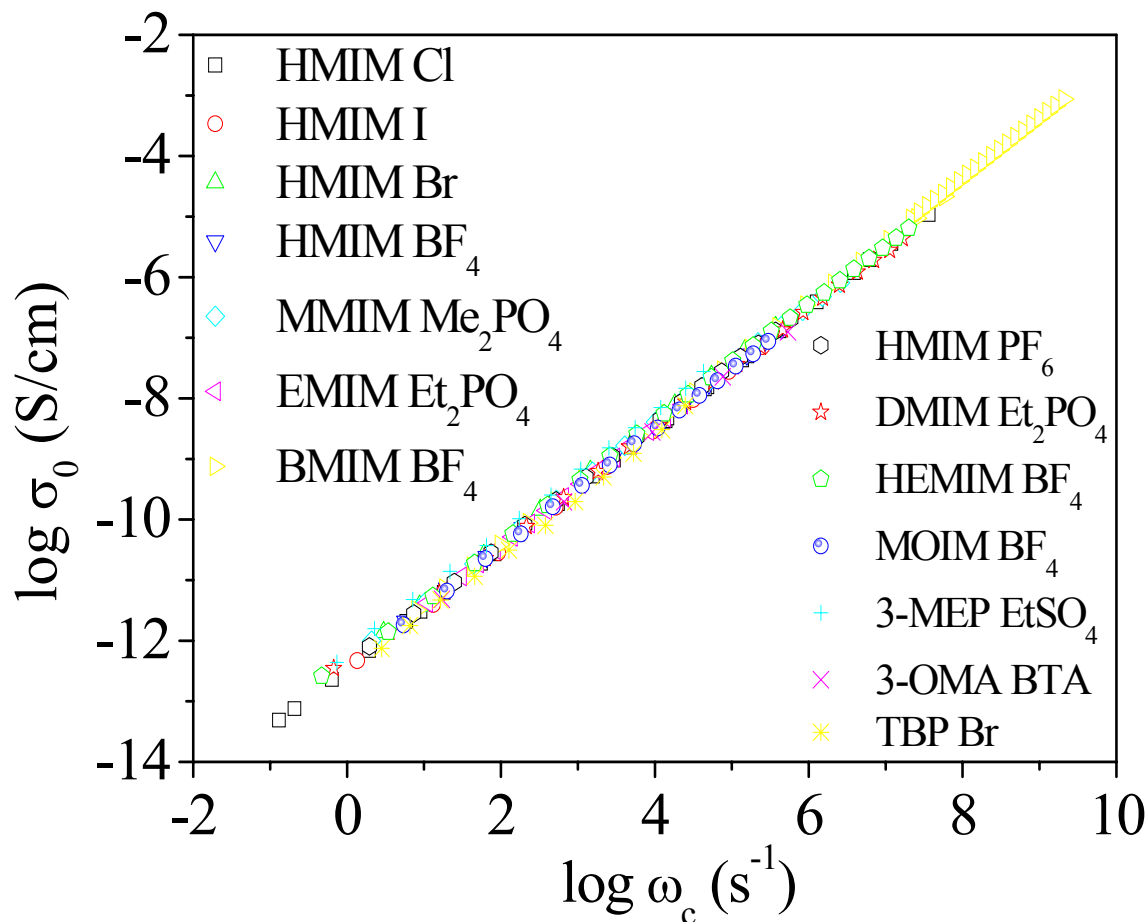
Excellent scaling with calorimetric glass transition temperature

Temperature dependence of characteristic diffusion rate



Characteristic diffusion rate shows a strong dependence on the anion (BF₄ < PF₆ < I < Br < Cl)

Universality of charge transport in ionic liquids



1. Universality despite significant structural variation (imidazolium-, ammonium-, pyridinium and phosphonium-based ionic liquids)

2. BNN relation holds

J. R. Sangoro et al. PCCP
DOI: 0.1039/b816106b,(2009)

.

Despite the structural variation, universal response is observed over 11 decades.

Predictions **checked** experimentally:

1.: $\omega_c = P \omega_\eta$

ω_c : characteristic (diffusion) rate

ω_η : structural relaxation rate

2.: $P = \frac{kT}{2\pi^2 G_\infty \lambda^2 a_s} \approx 1$

G_∞ : instantaneous shear modulus
($\sim 10^8$ Pa for ILs),

a_s : Stokes' hydrodynamic radius $\sim \lambda$,

k : Boltzmann constant,

λ : characteristic diffusion
length $\sim .2$ nm

3.: $D = \frac{\lambda^2 \omega_c}{2}$

D : molecular diffusion coefficient

4.: $\sigma_0 \sim \omega_c$

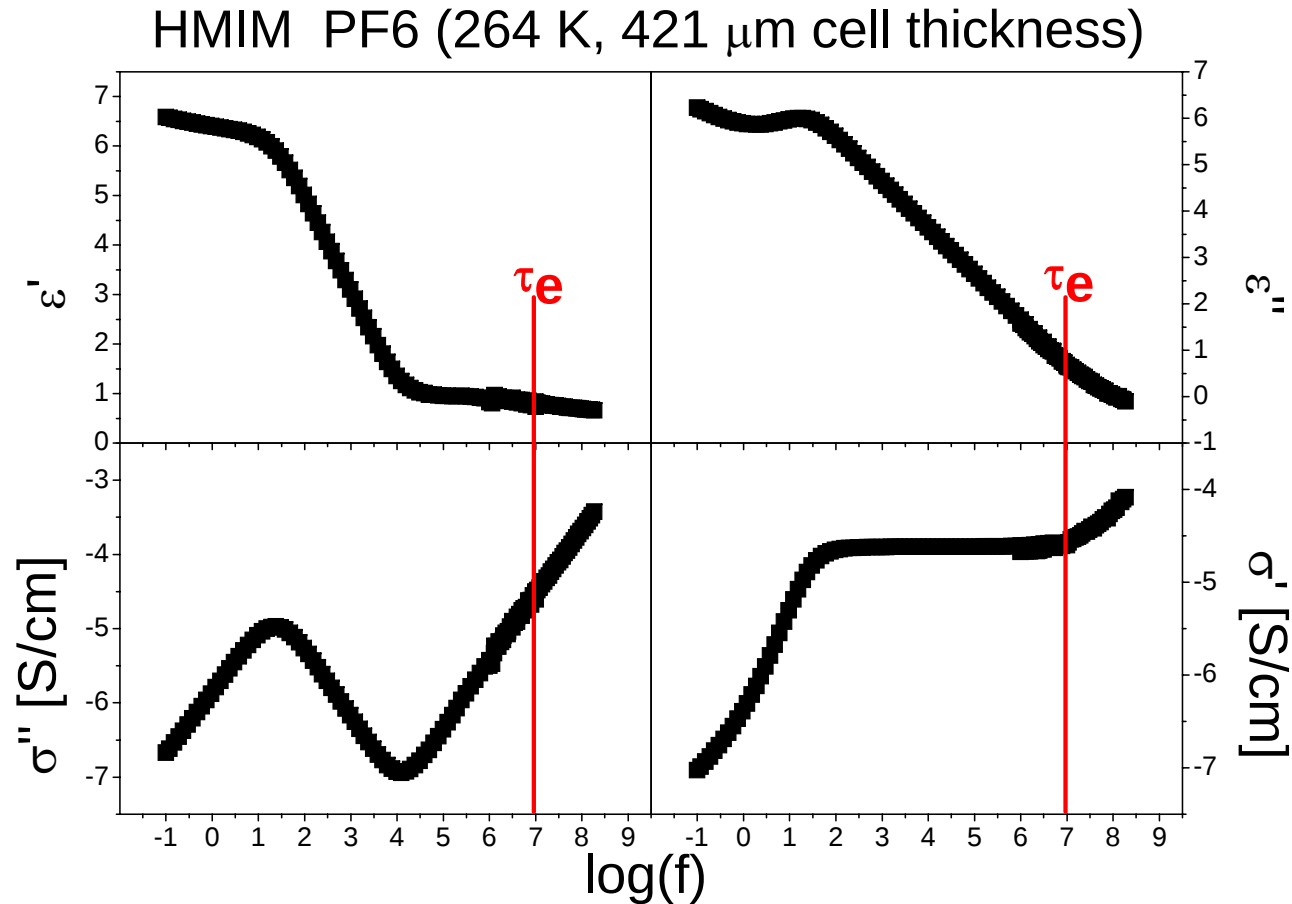
(Barton-Nishijima-Namikawa (BNN) relation)

Electrode polarization (EP) in Ionic Liquids (ILs) experimental features and *quantitative* description

Questions to be addressed:

1. What is the *signature of electrode polarisation (EP)* and how can it be analysed *quantitatively*?
2. What *quantitative information* can be deduced *from fits of the EP*?
3. What is the *quantitative model* to describe EP?
4. What *information can be deduced* from EP?

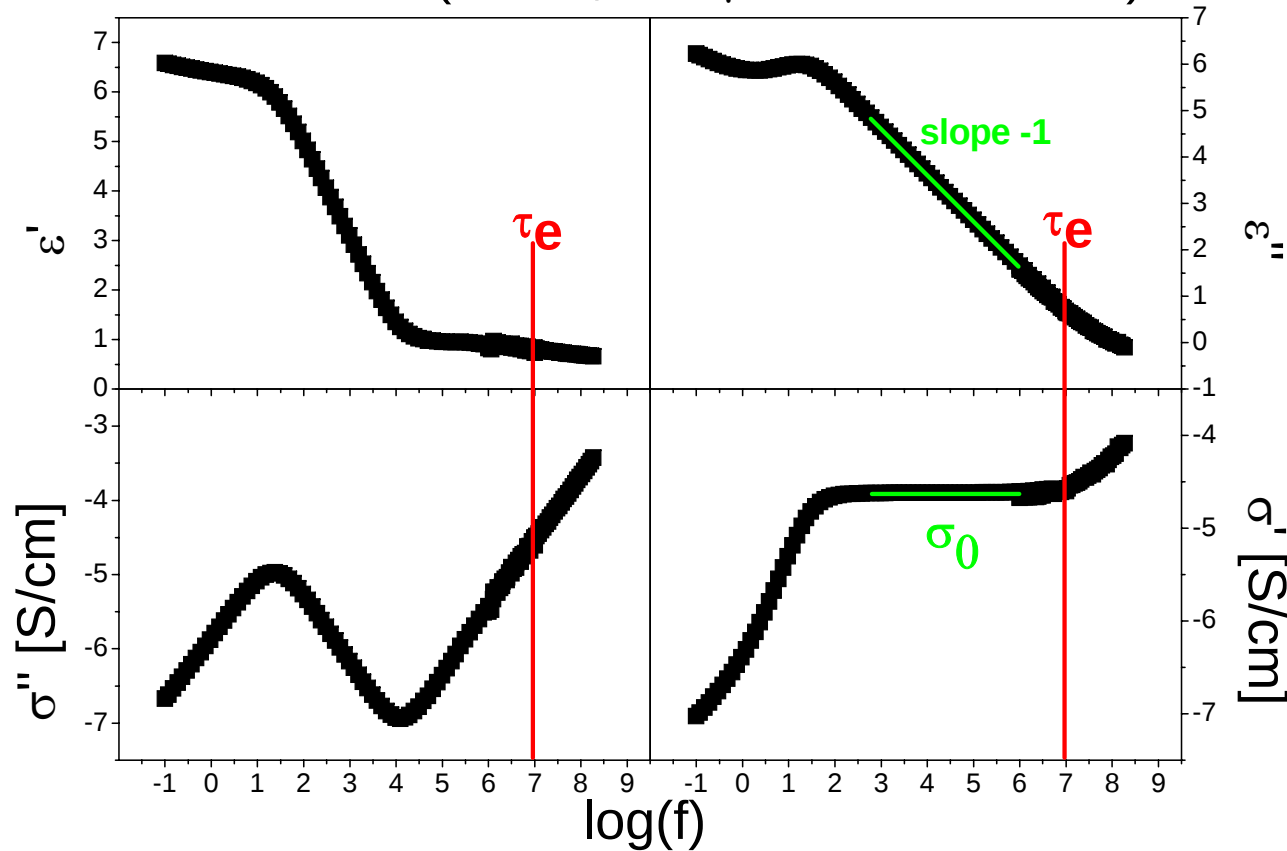
Experimental features of EP in ILs - frequency dependence



τ_e : hoping time

Experimental features of EP in ILs - frequency dependence

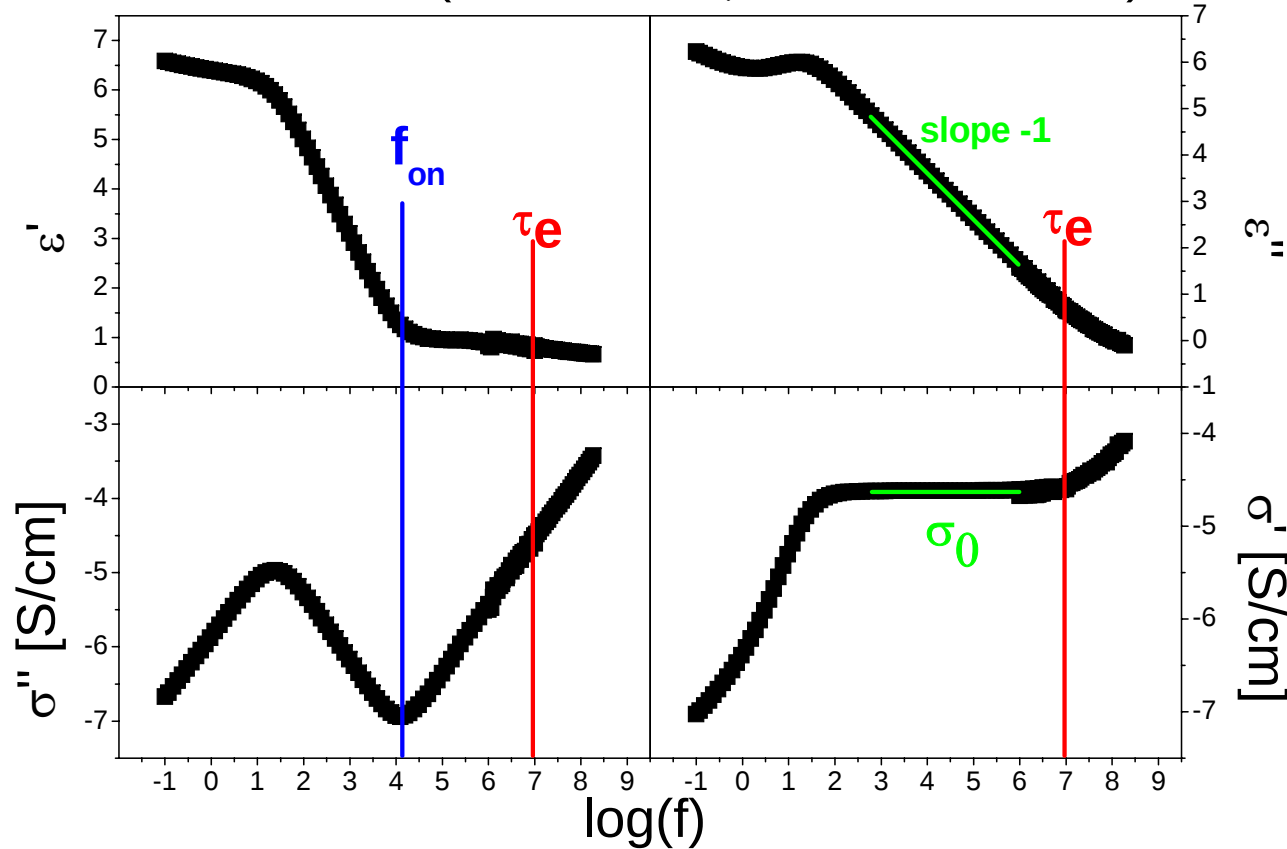
HMIM PF6 (264 K, 421 μm cell thickness)



τ_e : hopping time
 σ_0 : DC conductivity

Experimental features of EP in ILs - frequency dependence

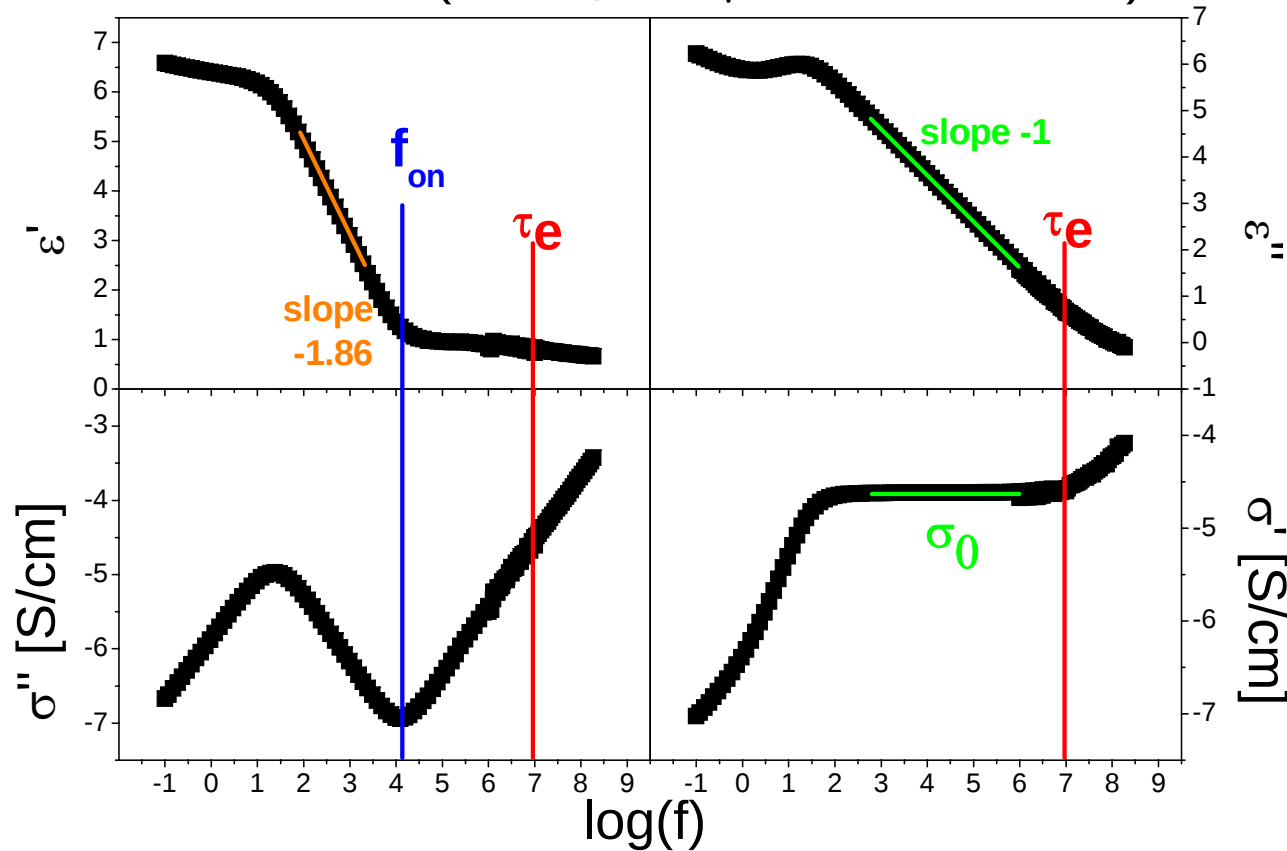
HMIM PF6 (264 K, 421 μm cell thickness)



τ_e : hoping time
 σ_0 : DC conductivity
 f_{on} : onset of electrode polarization

Experimental features of EP in ILs - frequency dependence

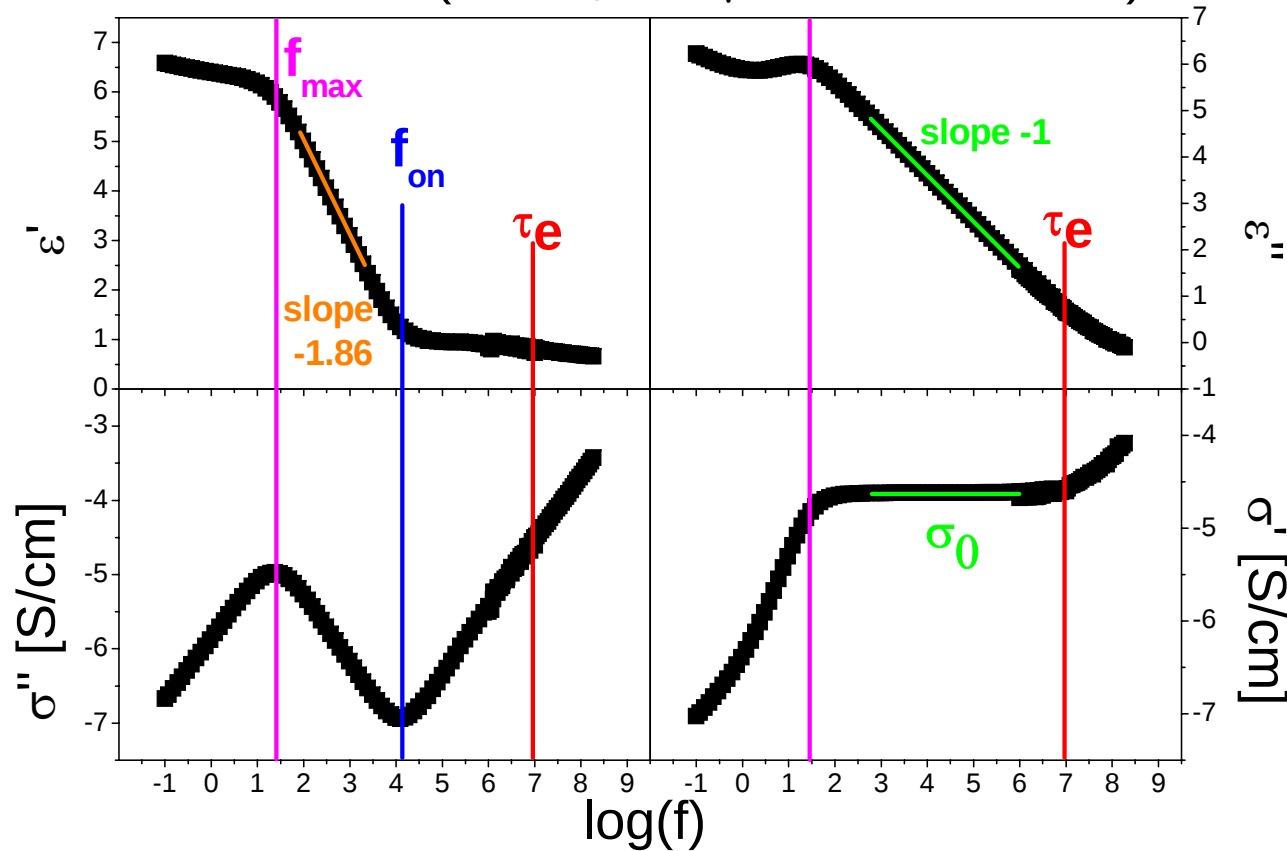
HMIM PF6 (264 K, 421 μm cell thickness)



τ_e : hoping time
 σ_0 : DC conductivity
 f_{on} : onset of electrode polarization
 slope of -1.86 in ϵ' for $f < f_{\text{on}}$

Experimental features of EP in ILs - frequency dependence

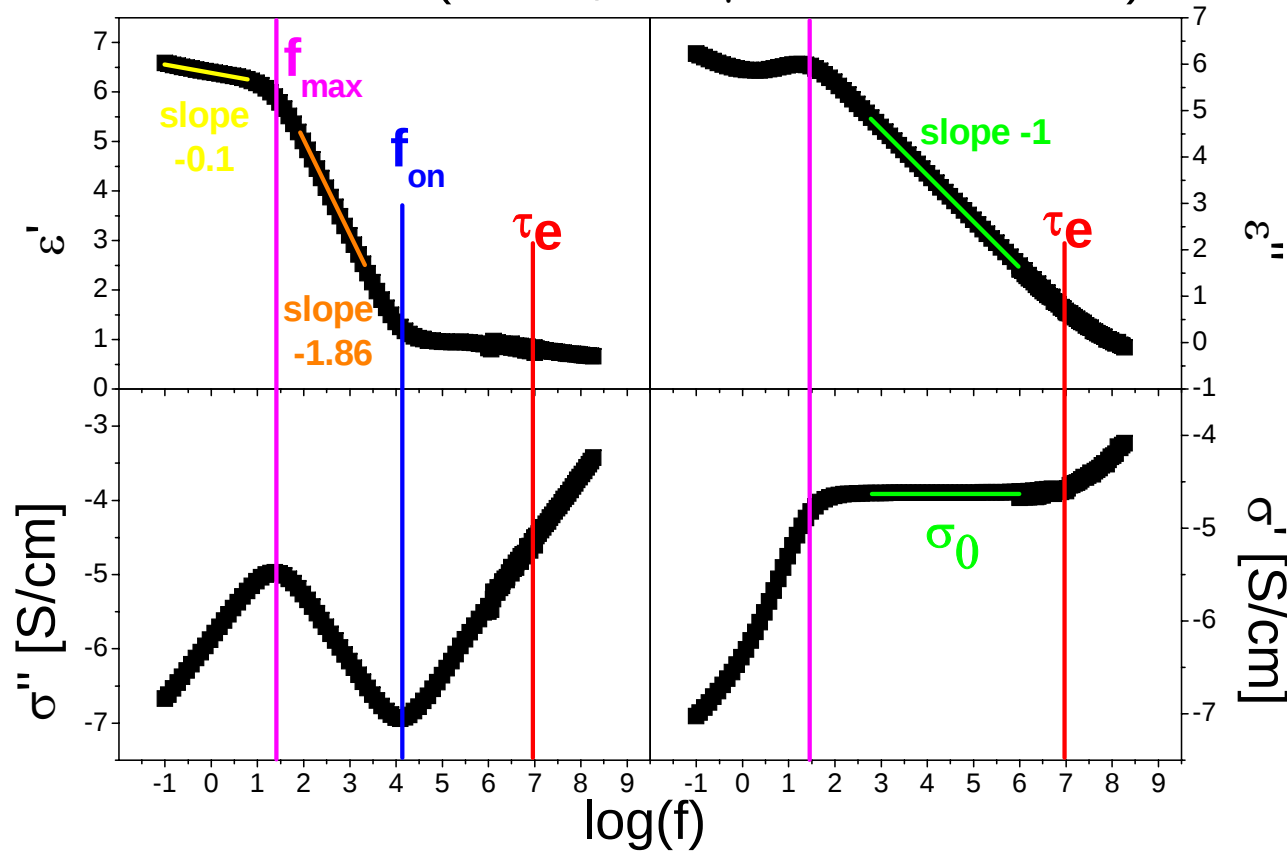
HMIM PF6 (264 K, 421 μm cell thickness)



τ_e : hoping time
 σ_0 : DC conductivity
 f_{on} : onset of electrode polarization
 slope of -1.86 in ϵ' for $f < f_{\text{on}}$
 f_{max} : full development of electrode polarization

Experimental features of EP in ILs - frequency dependence

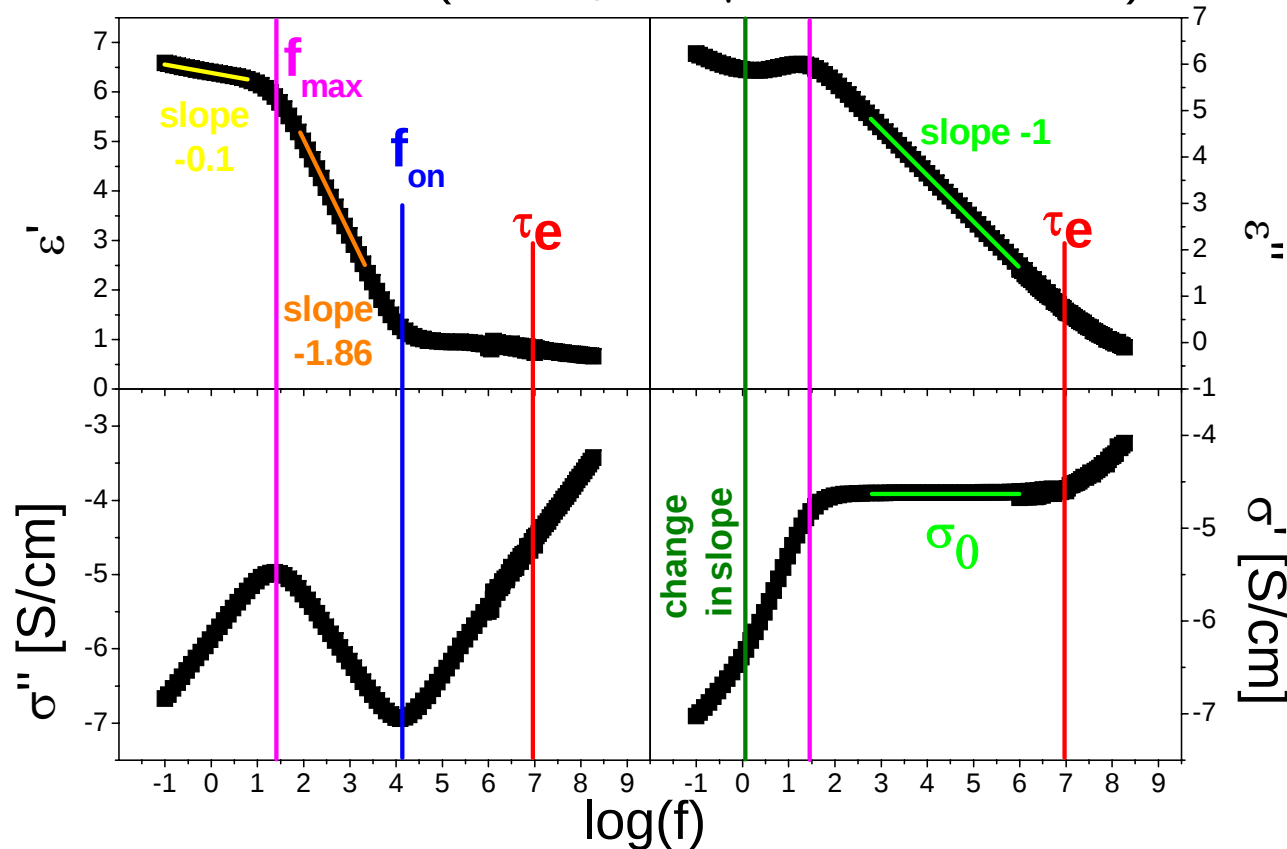
HMIM PF6 (264 K, 421 μm cell thickness)



- τ_e : hoping time
- σ_0 : DC conductivity
- f_{on} : onset of electrode polarization
- slope of -1.86 in ϵ' for $f < f_{\text{on}}$
- f_{max} : full development of electrode polarization
- slope of -0.1 in ϵ' for $f < f_{\text{max}}$

Experimental features of EP in ILs - frequency dependence

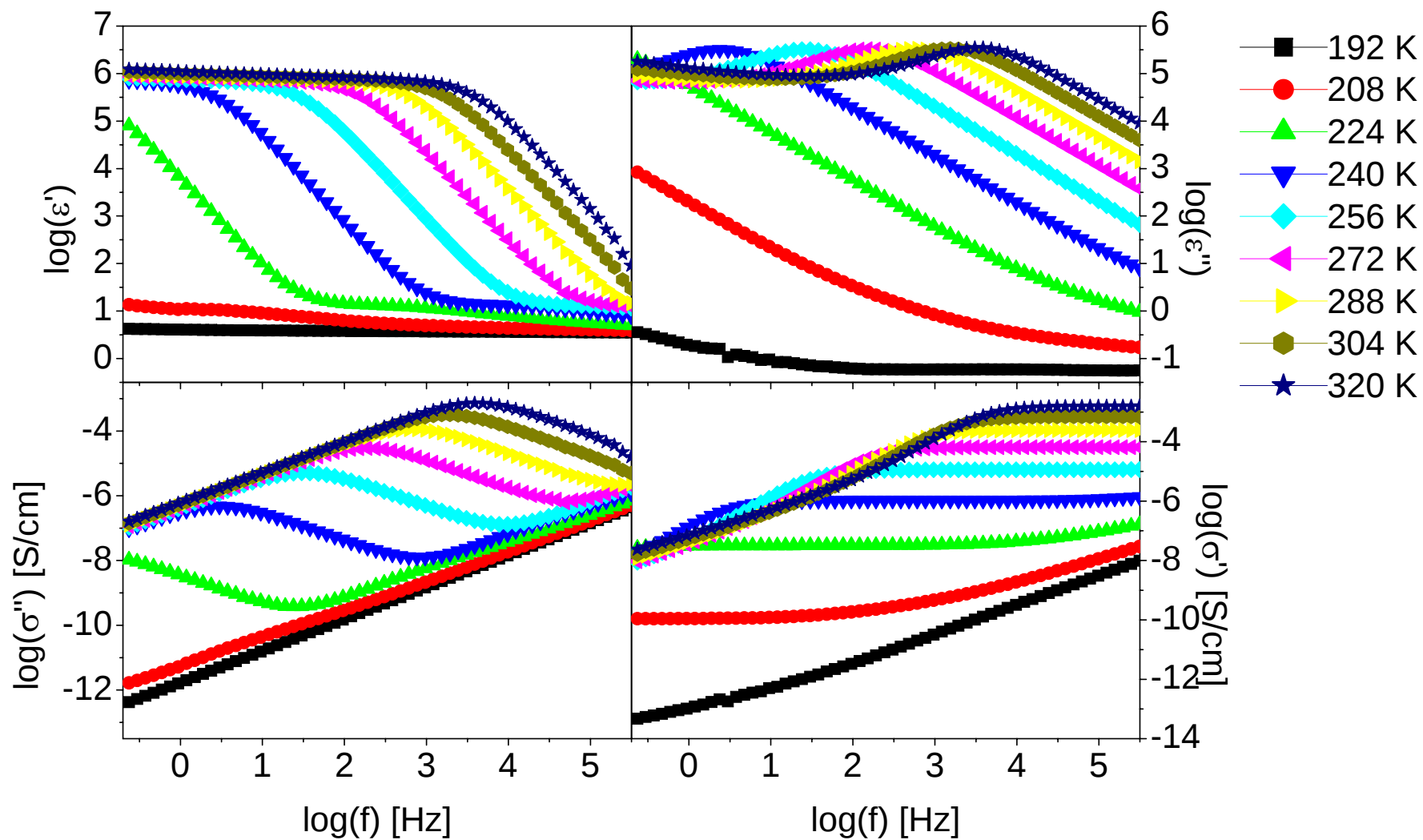
HMIM PF6 (264 K, 421 μm cell thickness)



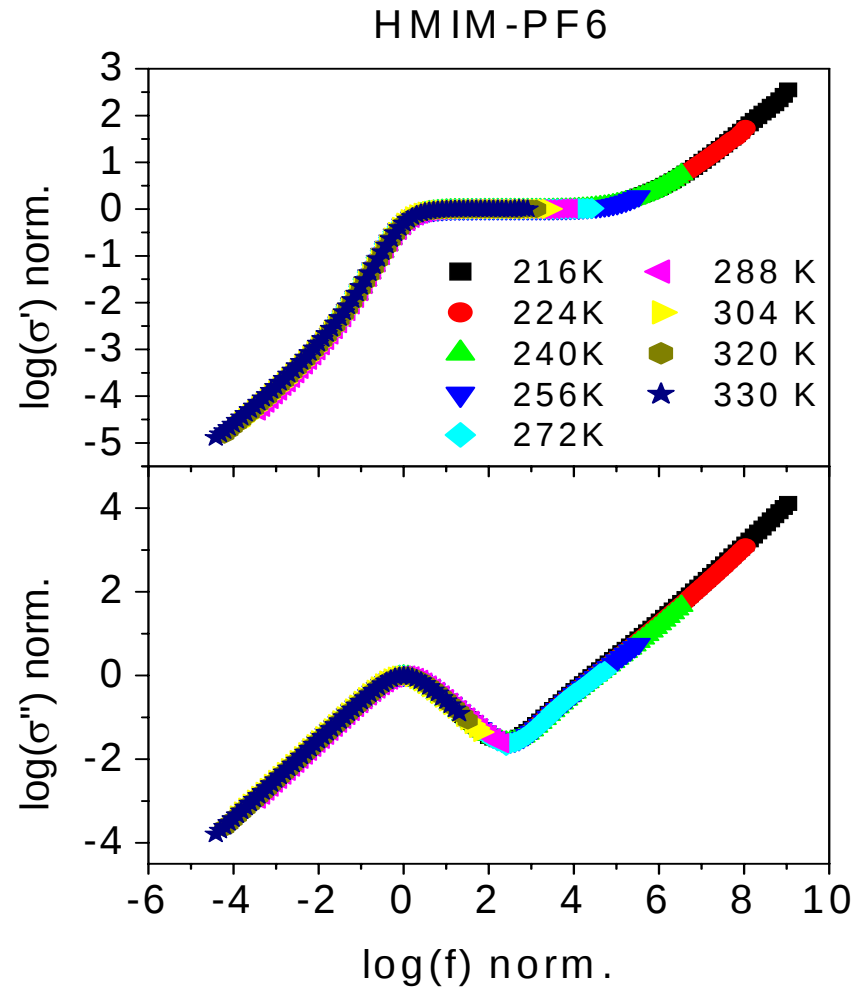
τ_e : hoping time
 σ_0 : DC conductivity
 f_{on} : onset of electrode polarization
 slope of -1.86 in ϵ' for $f < f_{\text{on}}$
 f_{max} : full development of electrode polarization
 slope of -0.1 in ϵ' for $f < f_{\text{max}}$
 change in slope of σ' for $f < f_{\text{max}}$

Dielectric spectra of ionic liquids - temperature dependence (experiment) -

HMIM-PF6



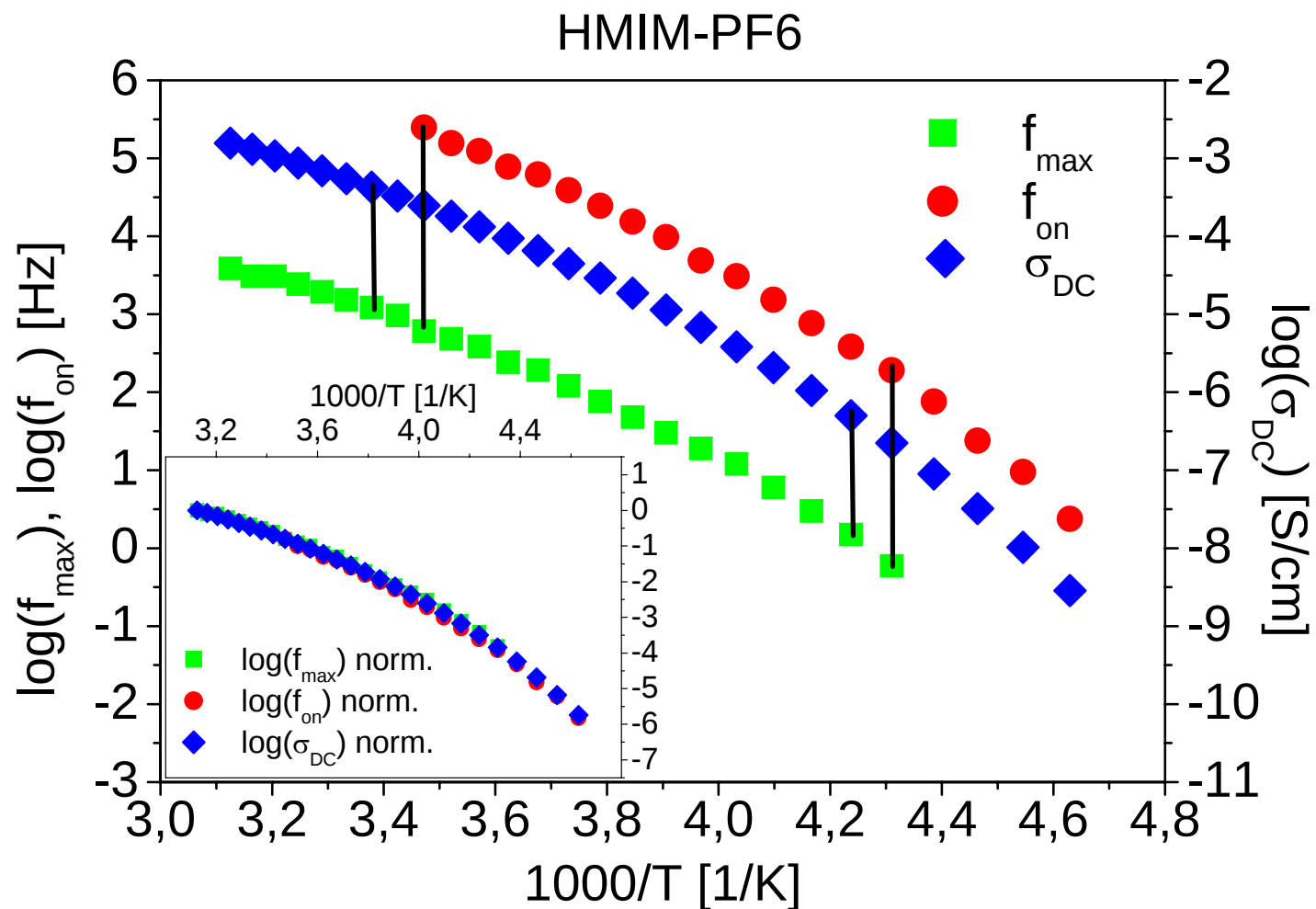
Dielectric spectra of ionic liquids - temperature dependence (experiment) -



**perfect scaling with temperature over many orders
of magnitude in time**

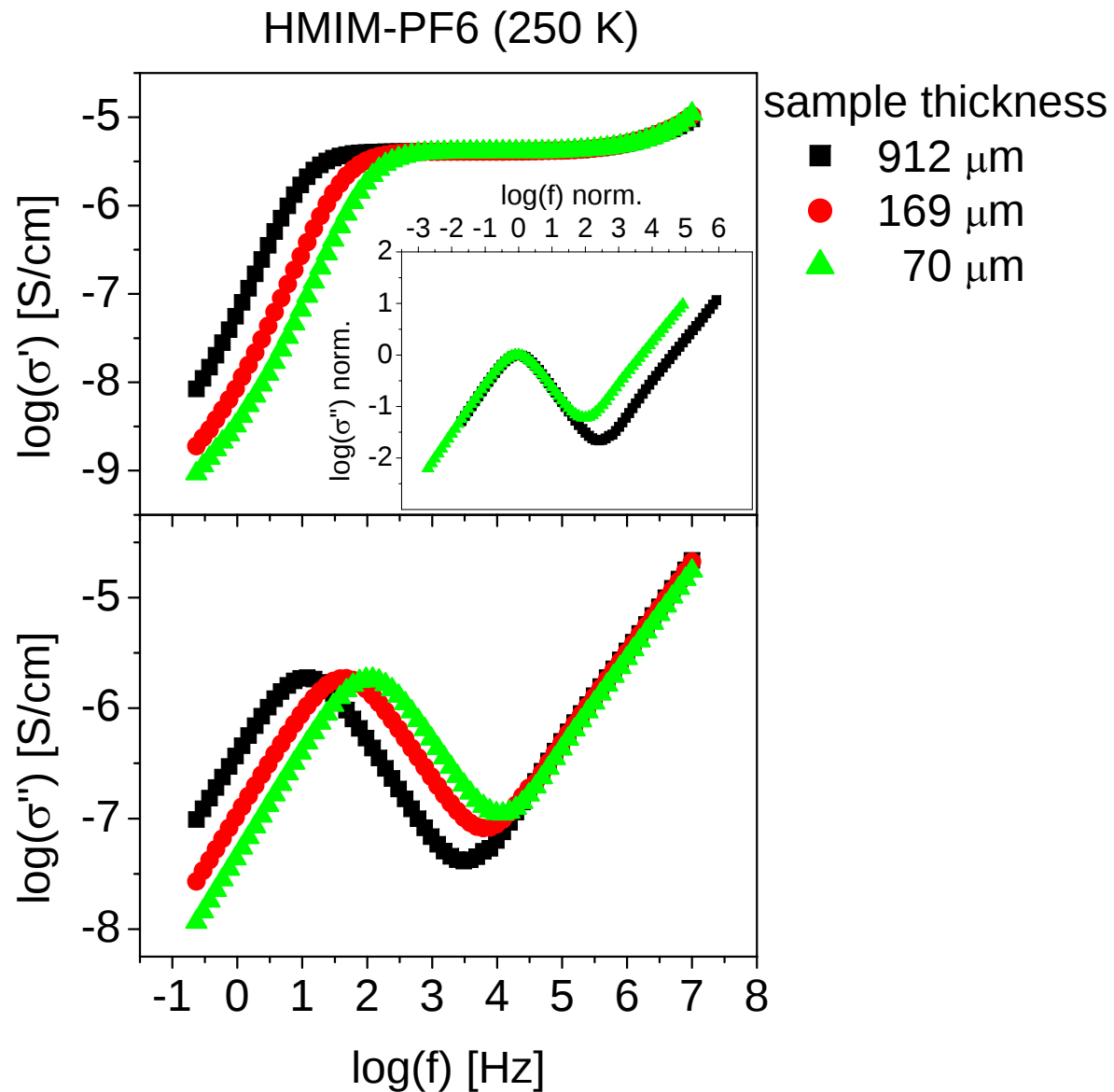
Dielectric spectra of ionic liquids

- temperature dependence (experiment) -

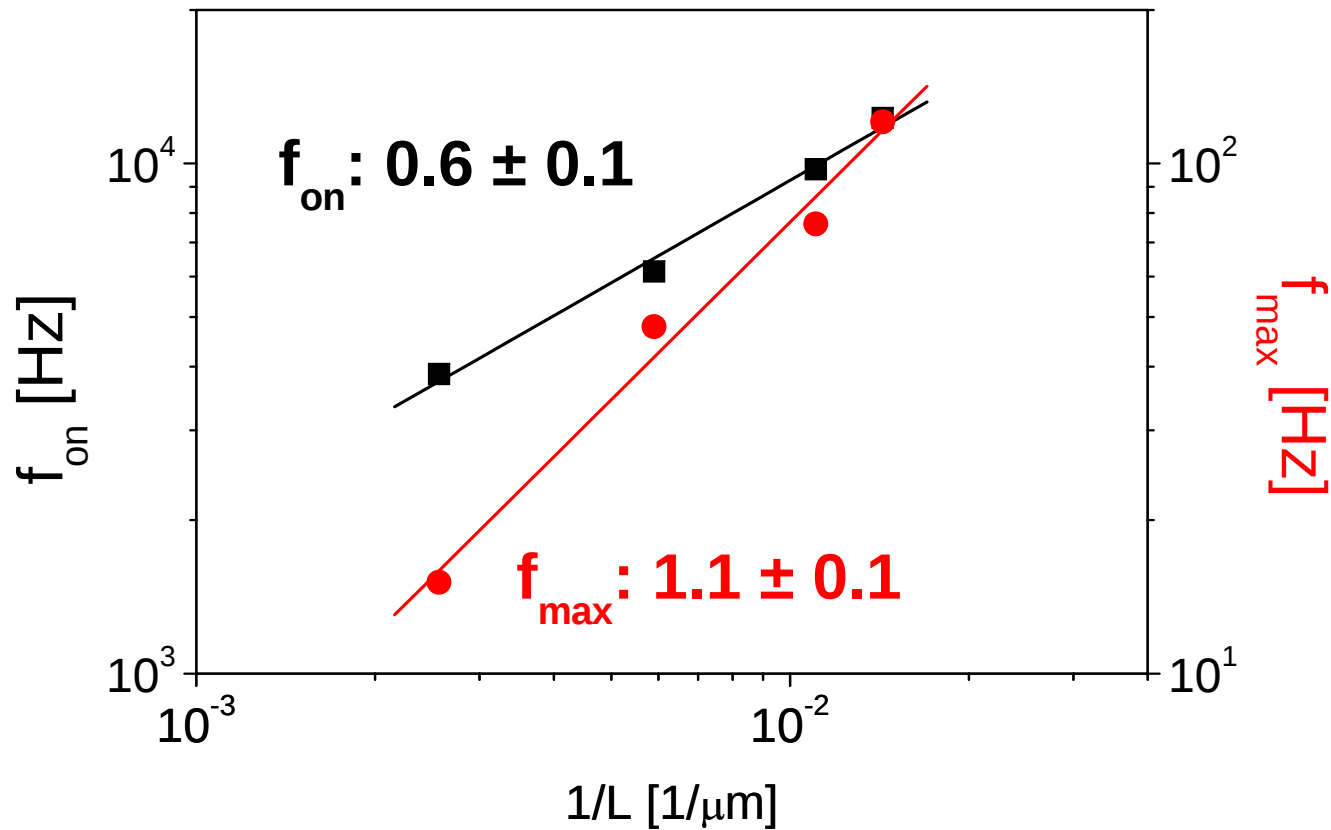


$f_{\max}$, f_{on} and σ_{DC} scale similarly with temperature

Dielectric spectra of ionic liquids- dependence on the **length of the sample cell** (experiment) -

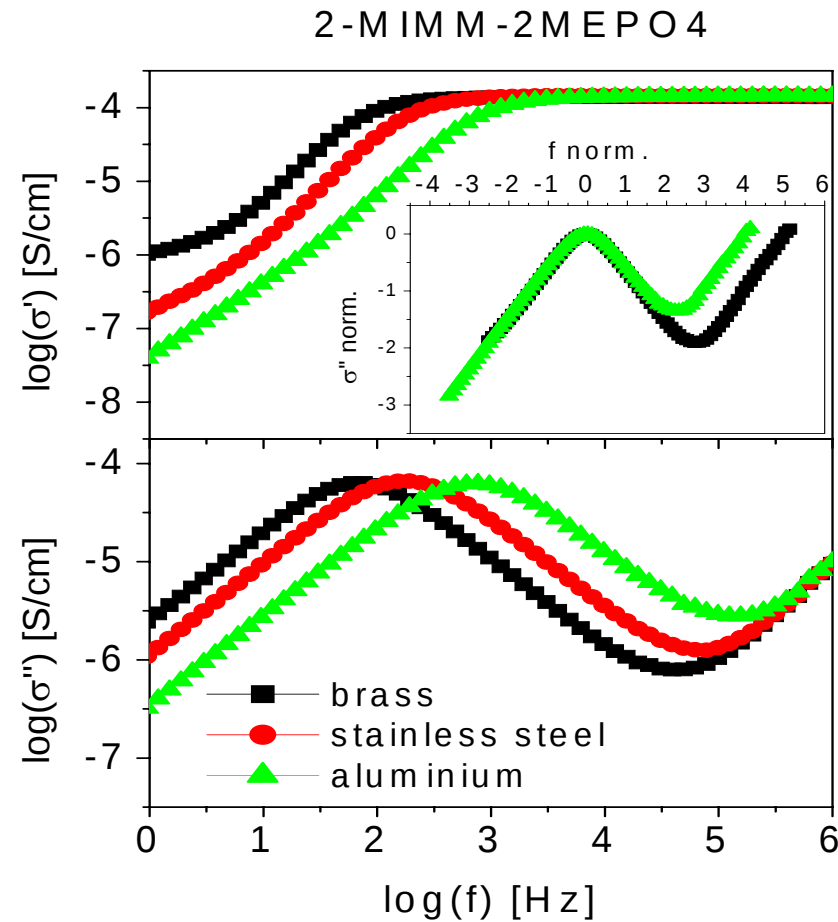


Dielectric spectra of ionic liquids- dependence on the length of the sample cell (experiment) -



No scaling of f_{on} and f_{max} with respect to $1/L$!

Dielectric spectra of ionic liquids – effect of the electrode material (experiment) -



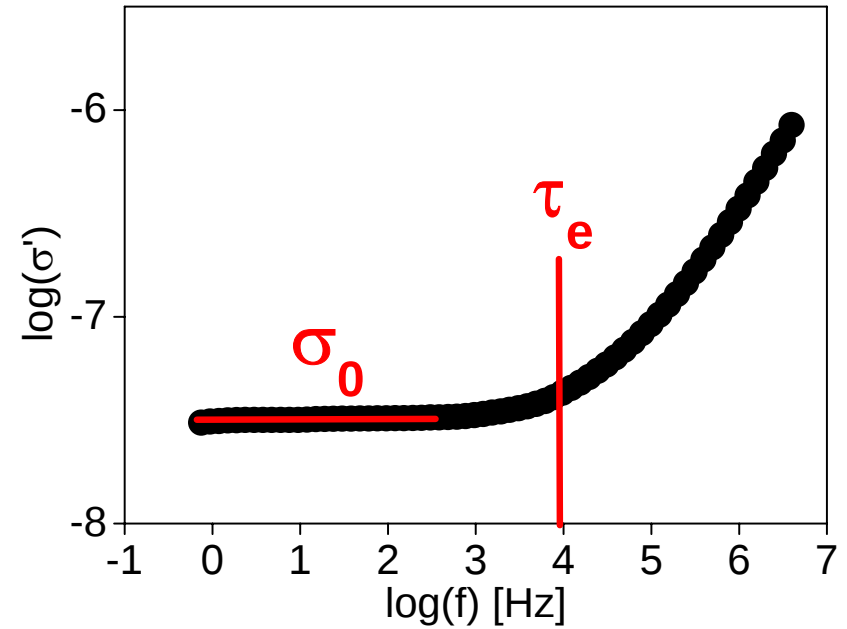
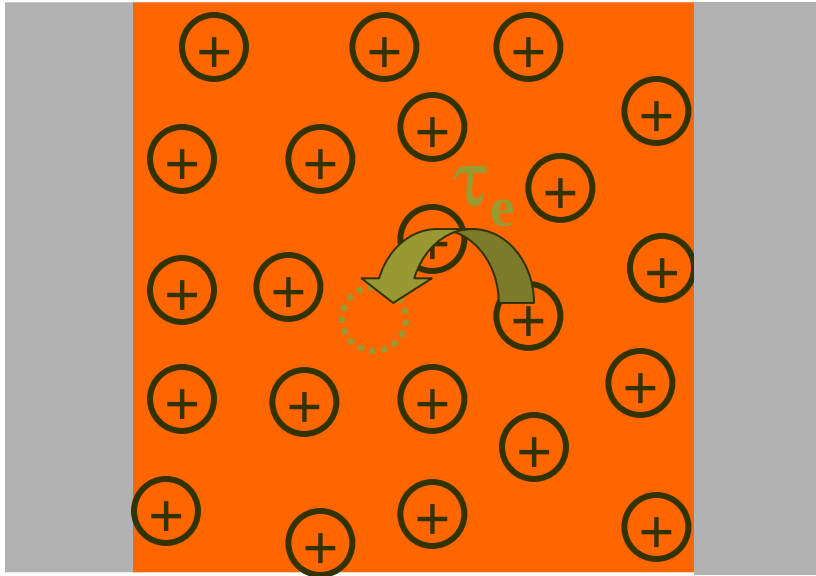
Pronounced effect of electrode material !

Experimental features of EP in ILs:

1. EP shows a complex dependence on frequency, temperature, concentration, sample length, material of the electrodes.
2. EP does *not* depend – below a certain (low) threshold, i.e. 10 V/cm – on the applied electric field.
3. EP does *not* depend on the roughness of the electrodes, as long as: roughness $\ll$ sample thickness
4. Over many decades in frequency, scaling with respect to variation of temperature and concentration
5. No scaling with respect to variation of the length of the sample cell and material of the electrodes.

What is the microscopic mechanism of EP ?

Charge transport in the bulk

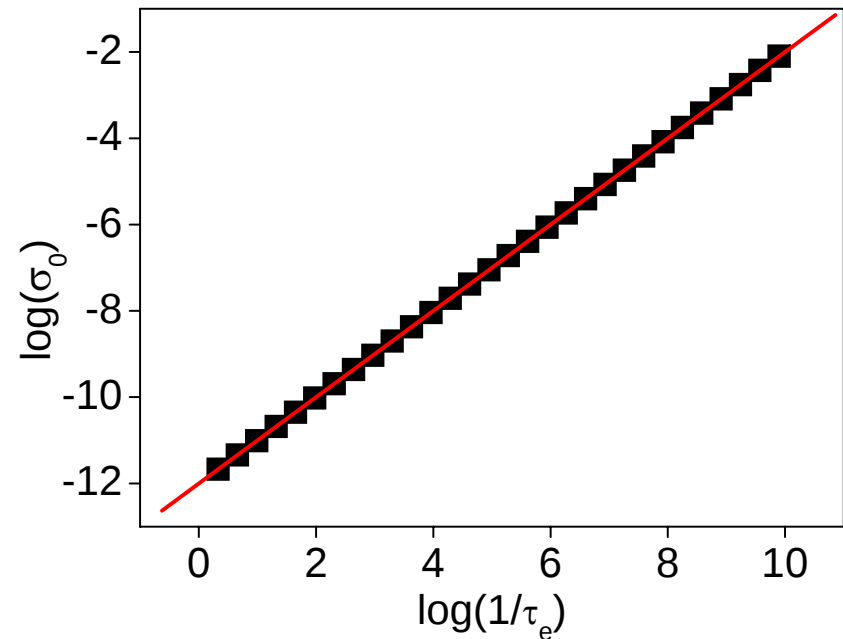


1. hopping time in bulk:

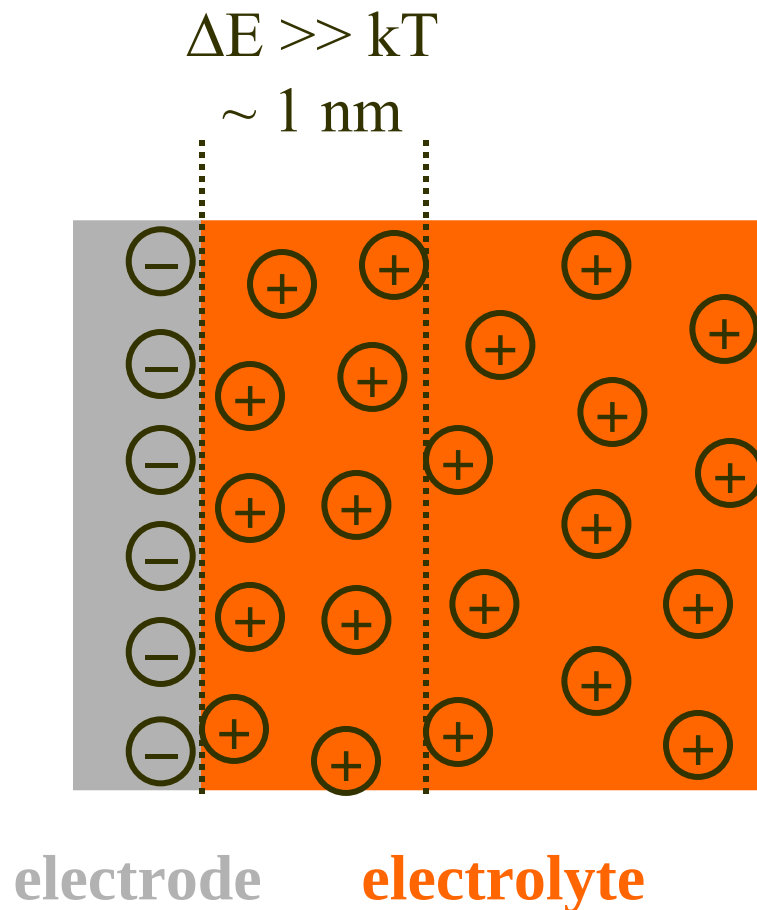
$$\tau_e = \tau_0 \exp\left(\frac{E}{kT}\right)$$

2. BNN-relation:

$$\sigma_0 \sim \frac{1}{\tau_e}$$



Charge transport at interfaces



Due to the **coulombic interactions increase by many orders of magnitude** in the hoping time at the interface.

hoping time at the interface:

$$E_i \cong E_b + \Delta E$$

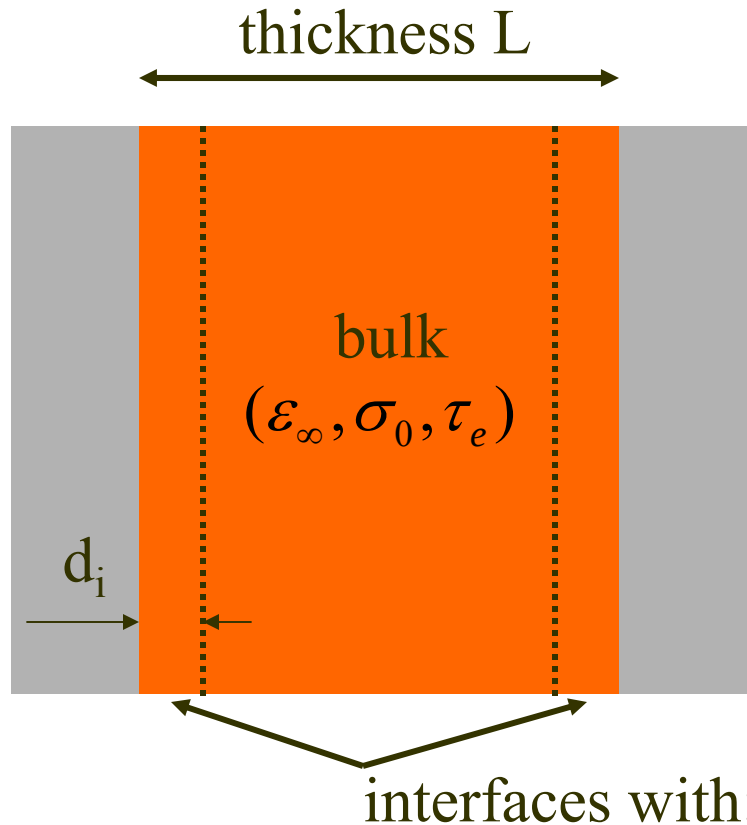
$$\frac{\tau_e(\text{interface})}{\tau_e(\text{bulk})} \approx \exp\left(\frac{\Delta E}{kT}\right) \gg 1$$

Since: $\sigma_0 \sim \frac{1}{\tau_e}$ (BNN relation)

this implies:

$$\sigma_0(\text{interface}) \ll \sigma_0(\text{bulk})$$

Microscopic model of sample cell



$$\frac{L}{\epsilon_{meas}^*} = \frac{2d_i}{\epsilon_i^*} + \frac{L - 2d_i}{\epsilon_b^*}$$

$$\epsilon_b^* = f\left(\epsilon_{\infty}, \sigma_0^{\text{bulk}}, \tau_0^{\text{bulk}}\right)$$

$$\epsilon_i^* = f\left(\epsilon_{\infty}, \sigma_0^i, \tau_0^i\right)$$

$$\frac{\tau_e(\text{interface})}{\tau_e(\text{bulk})} \approx \frac{\sigma_0(\text{bulk})}{\sigma_0(\text{interface})} \gg 1$$

Electrode polarization

$$\frac{L}{\varepsilon_{meas}^*} = \frac{2d_i}{\varepsilon_i^*} + \frac{L-2d_i}{\varepsilon_b^*}$$

$$\varepsilon'_{meas} = (x+1) \left((\varepsilon'_b)^2 + (\varepsilon''_b)^2 \right) \frac{x\varepsilon'_b + y\varepsilon'_i}{(x\varepsilon'_b + y\varepsilon'_i)^2 + (x\varepsilon''_b + y\varepsilon''_i)^2}$$

$$\varepsilon''_{meas} = (x+1) \left((\varepsilon'_b)^2 + (\varepsilon''_b)^2 \right) \frac{x\varepsilon''_b + y\varepsilon''_i}{(x\varepsilon'_b + y\varepsilon'_i)^2 + (x\varepsilon''_b + y\varepsilon''_i)^2}$$

$$\text{where: } x = \frac{L-2d_i}{2d_i} \quad y = \frac{(\varepsilon'_b)^2 + (\varepsilon''_b)^2}{(\varepsilon'_i)^2 + (\varepsilon''_i)^2}$$

with

$$\varepsilon^* = \frac{\sigma^*}{i\varepsilon_0\omega} \quad \text{and} \quad \sigma^*(\omega) = \sigma_0 \left[\frac{i\omega\tau_e}{\ln(1+i\omega\tau_e)} \right] \quad \text{Dyre functions}$$

EP - consequences of the model

$$\frac{d}{d\omega}(\sigma''_{\text{meas}}) = 0 \quad \Rightarrow$$

$$f_{\text{on}} \cong \frac{\sigma_0}{\varepsilon_0} \frac{1}{\sqrt{\varepsilon'_b \varepsilon'_i}} \sqrt{\frac{2d_i}{L}}$$

$$f_{\text{max}} \cong \frac{\sigma_0}{\varepsilon_0} \frac{1}{2\pi\varepsilon'_i} \frac{2d_i}{L}$$

Consequences:

$$1. f_{\text{on}} \sim f_{\text{max}} \sim \sigma_0$$

$$2. f_{\text{on}} \sim \frac{1}{\sqrt{L}}, \text{ while } f_{\text{max}} \sim \frac{1}{L}$$

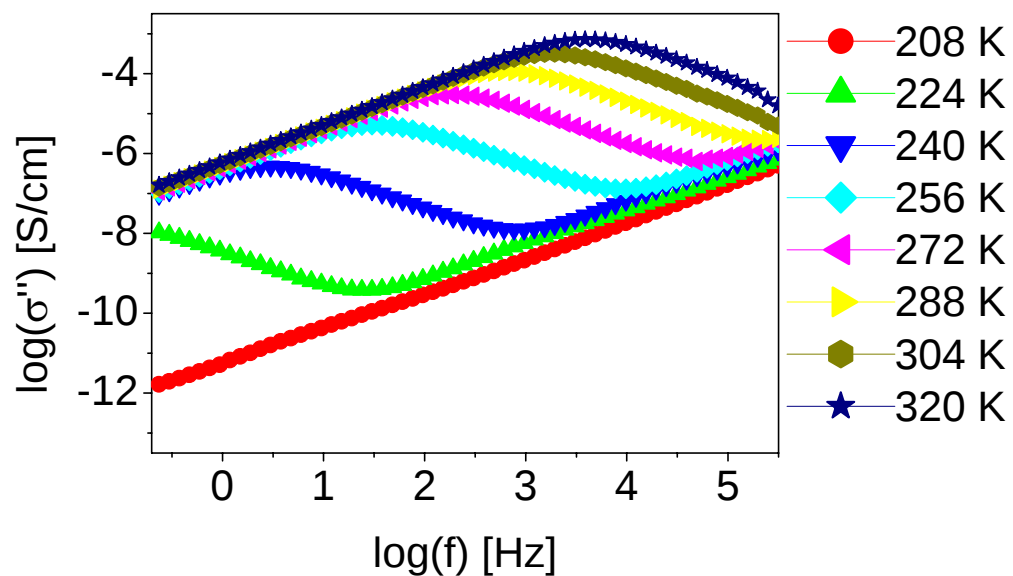
$$3. f_{\text{on}} \sim \sqrt{d_i}, \text{ while } f_{\text{max}} \sim d_i$$

$$4. f = f_{\text{max}} \text{ solution for both } \frac{d(\sigma''_{\text{meas}})}{d\omega} = 0 \text{ and } \frac{d(\varepsilon''_{\text{meas}})}{d\omega} = 0$$

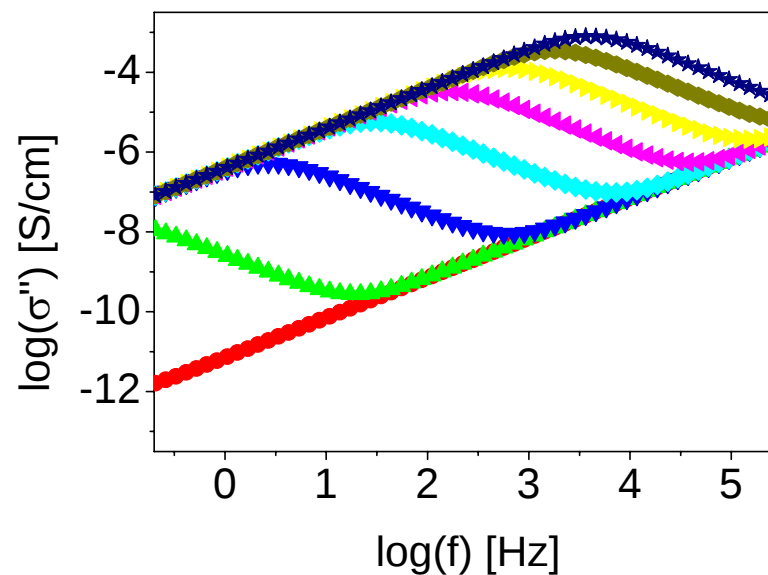
$$5. \sigma_0 = 2\pi\varepsilon_0\varepsilon_s \frac{(f_{\text{on}})^2}{f_{\text{max}}}$$

Comparison of experiment and calculations - dielectric spectra of ionic liquids

experiment

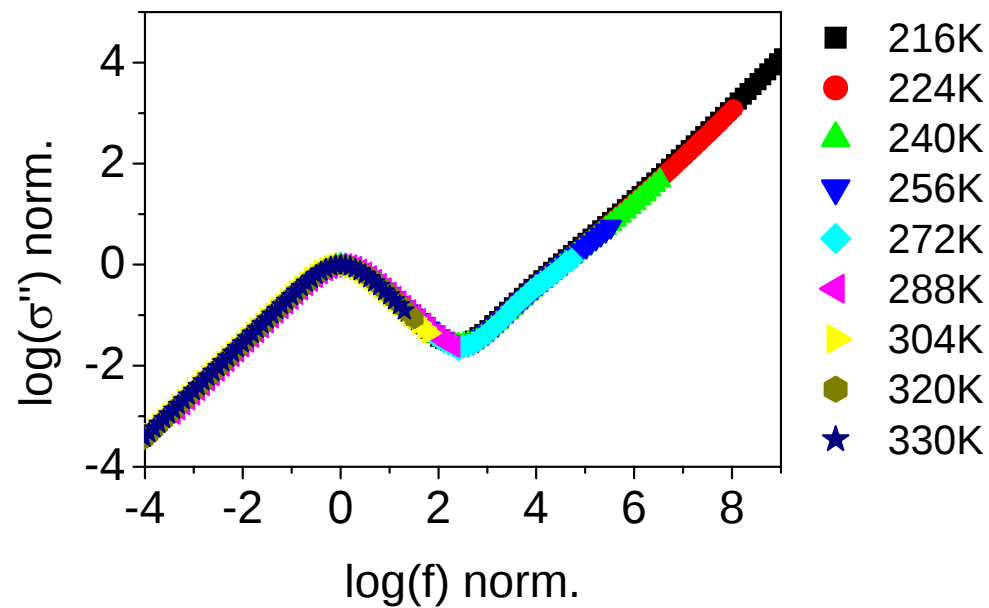


calculations

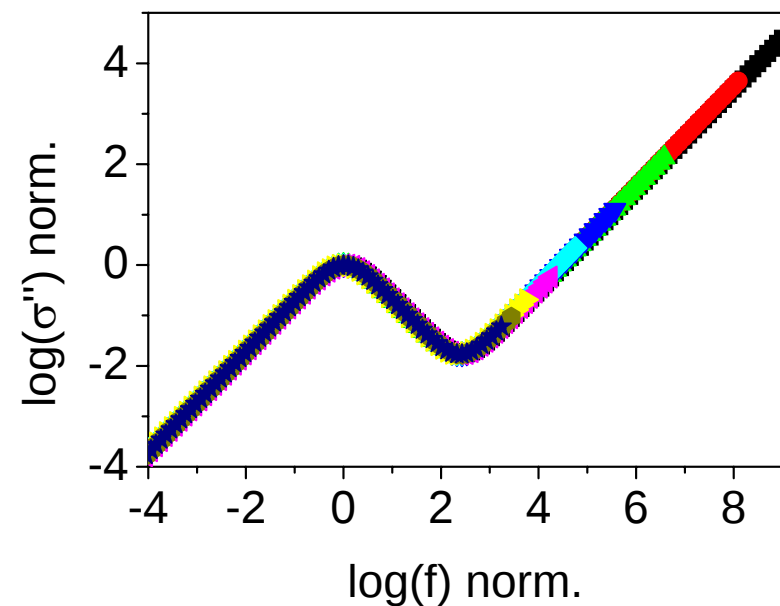


Comparison of experiment and calculations - dielectric spectra of ionic liquids

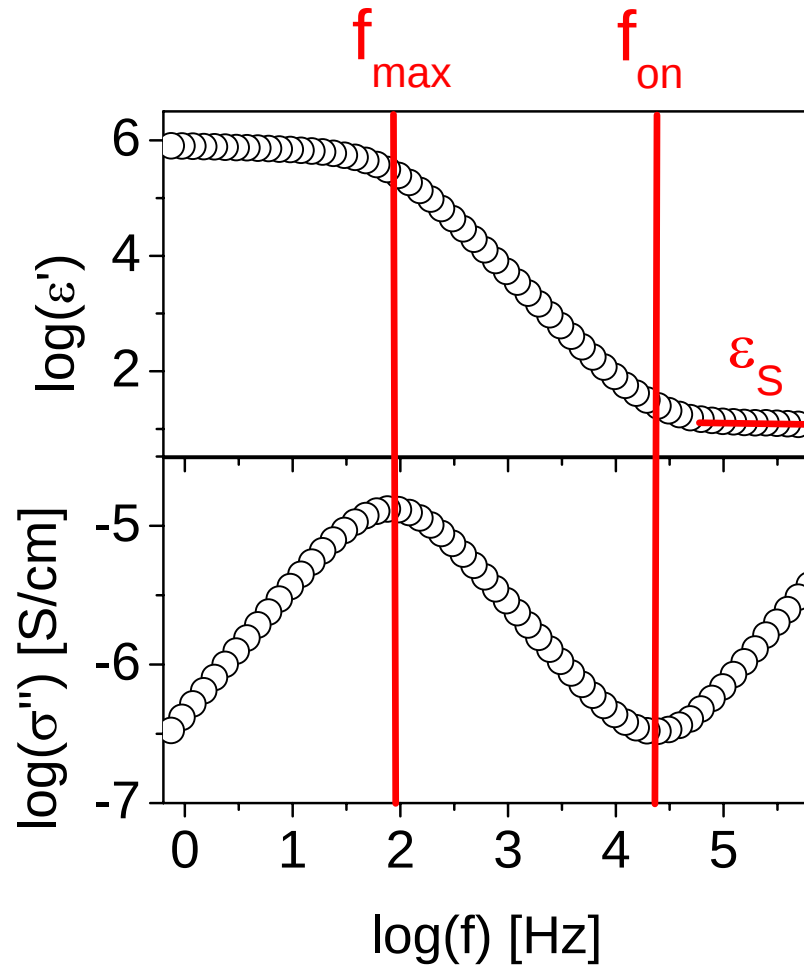
experiment



calculations



What information can be deduced from EP – a novel formula.



$$\sigma_0 = ?$$

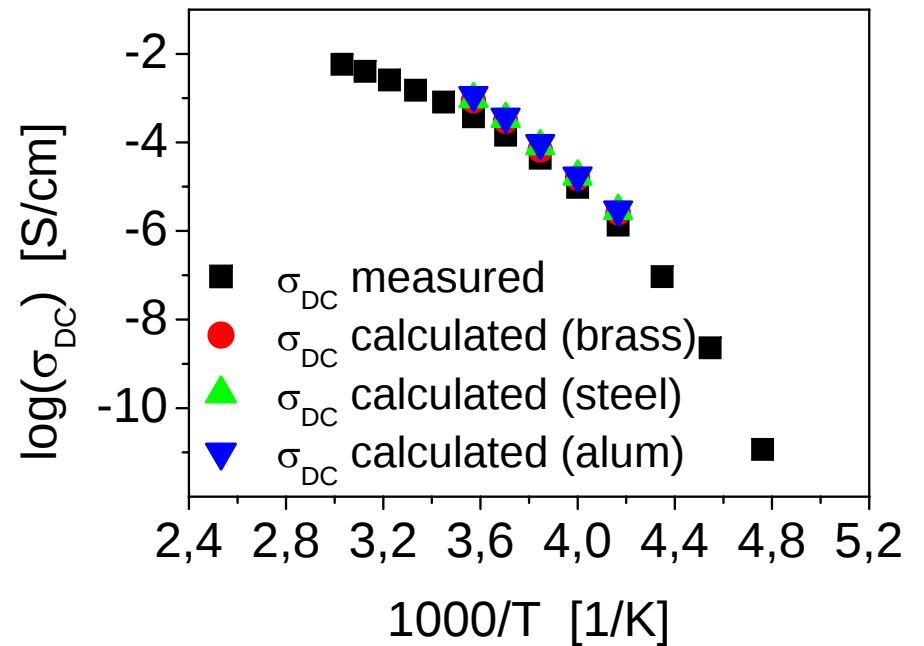
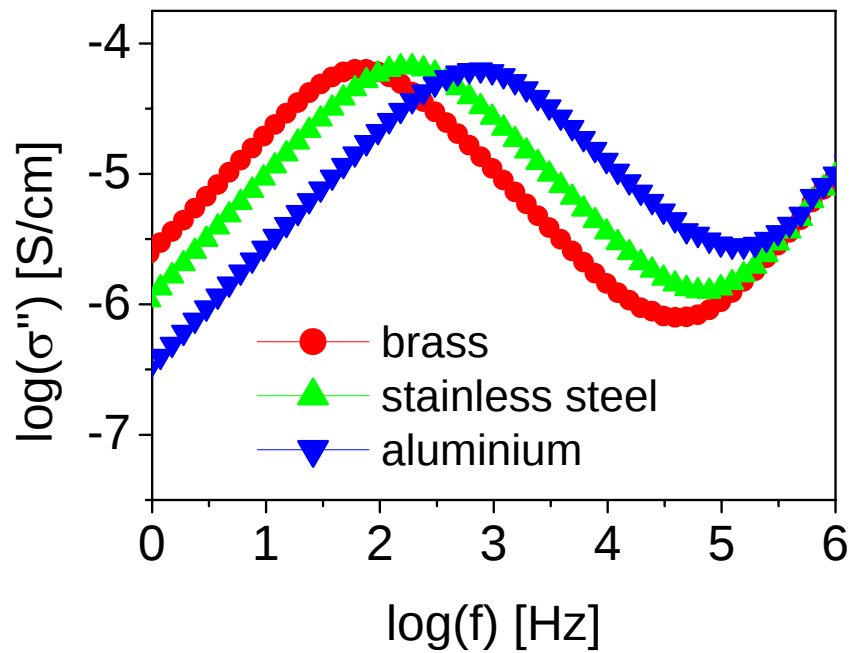
$$\sigma_0 = 2\pi\epsilon_0\epsilon_s \frac{(f_{\text{on}})^2}{f_{\text{max}}}$$

discovered by Anatoli Serghei et al.

Test of the validity of the novel formula

$$\sigma_0 = 2\pi\epsilon_0\epsilon_s \frac{(f_{\text{on}})^2}{f_{\text{max}}}$$

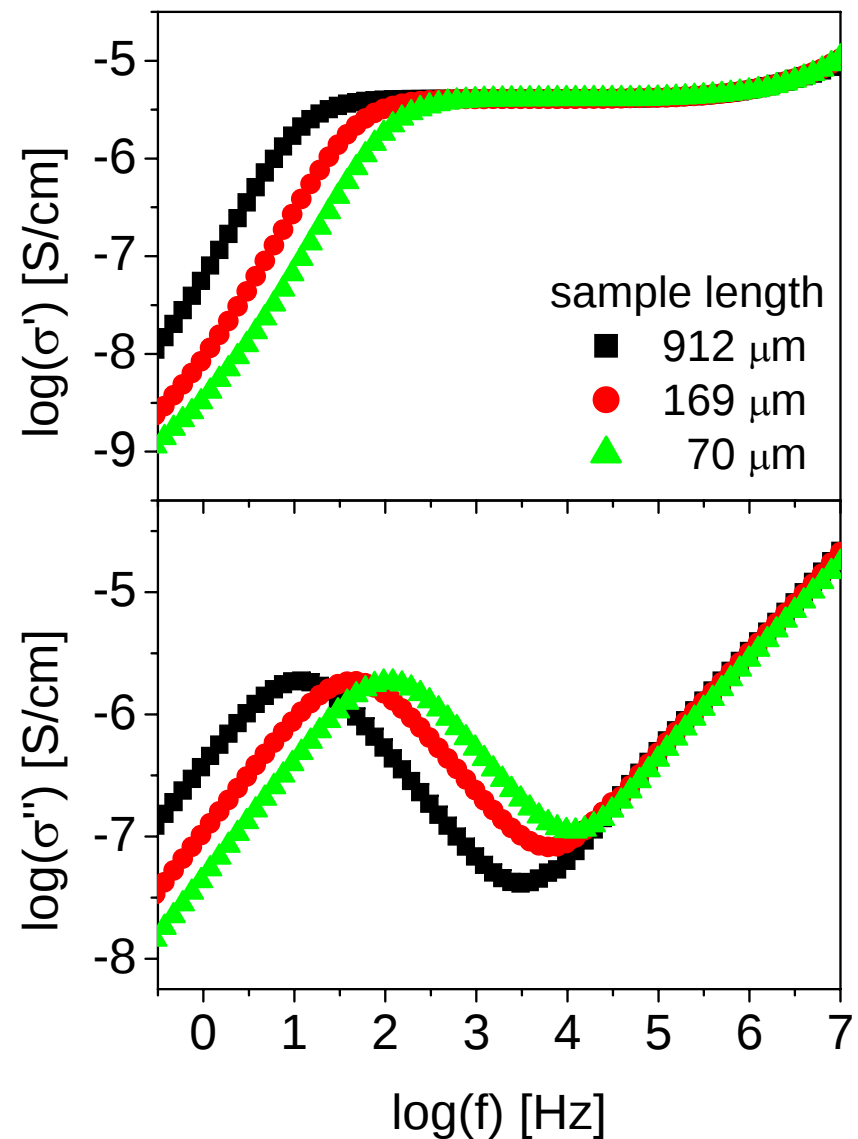
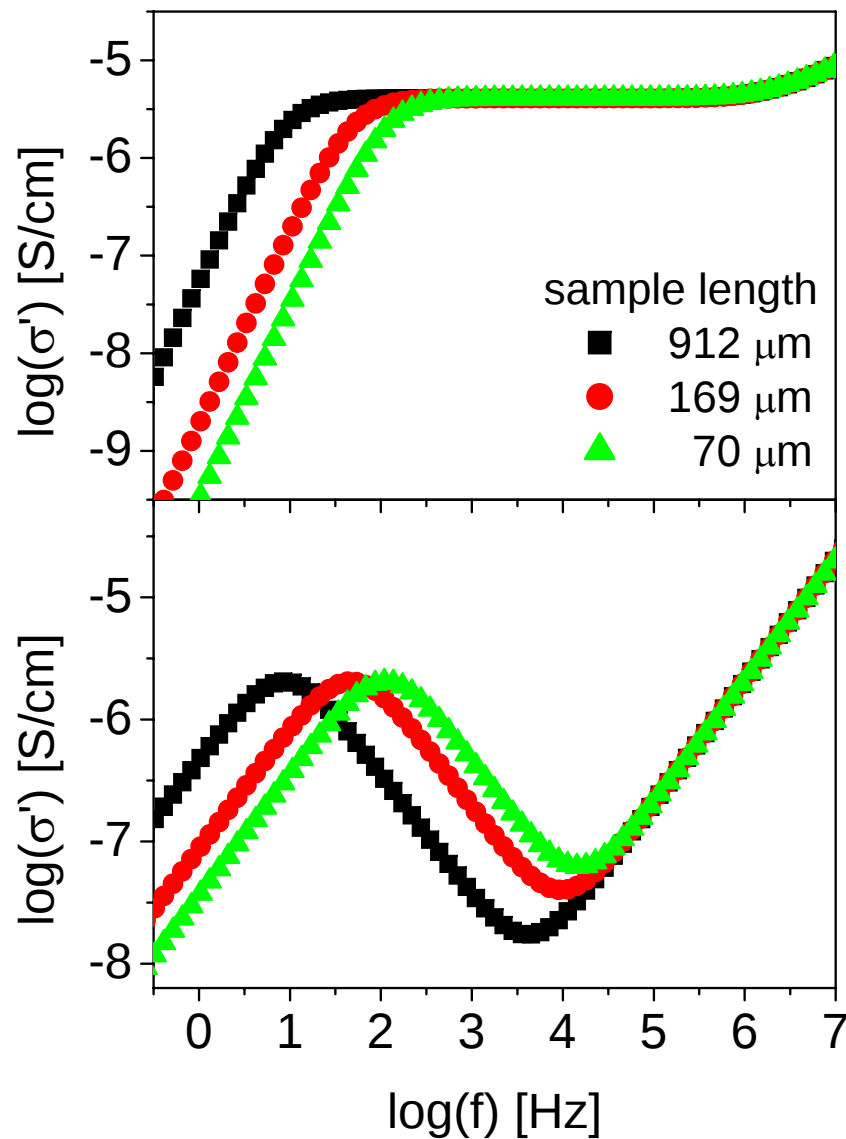
2-MIMM-2MEPO4



Dielectric spectra of ionic liquids - **length dependence** (2MIMM-2MePO₄, experiment and calculations) -

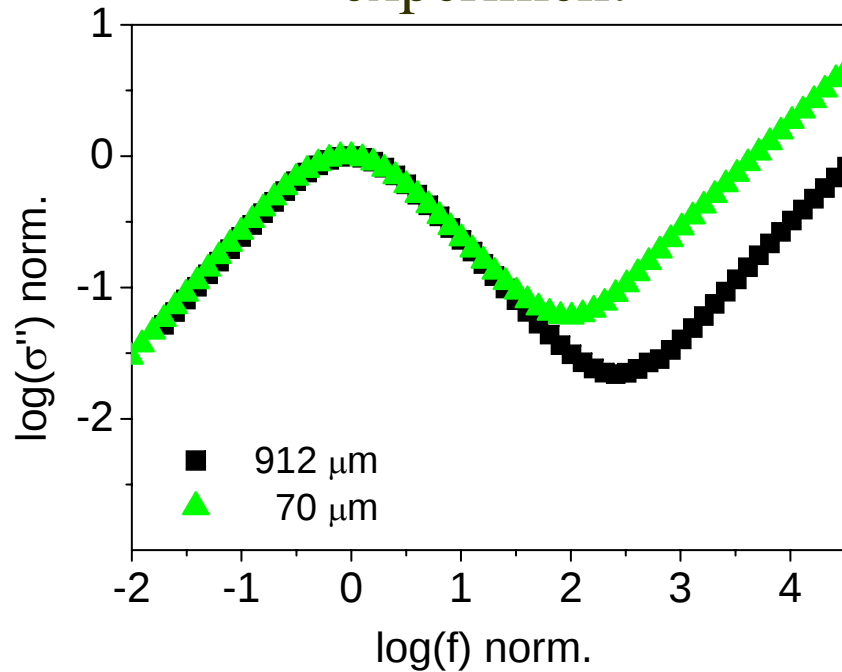
Calculations

experiment

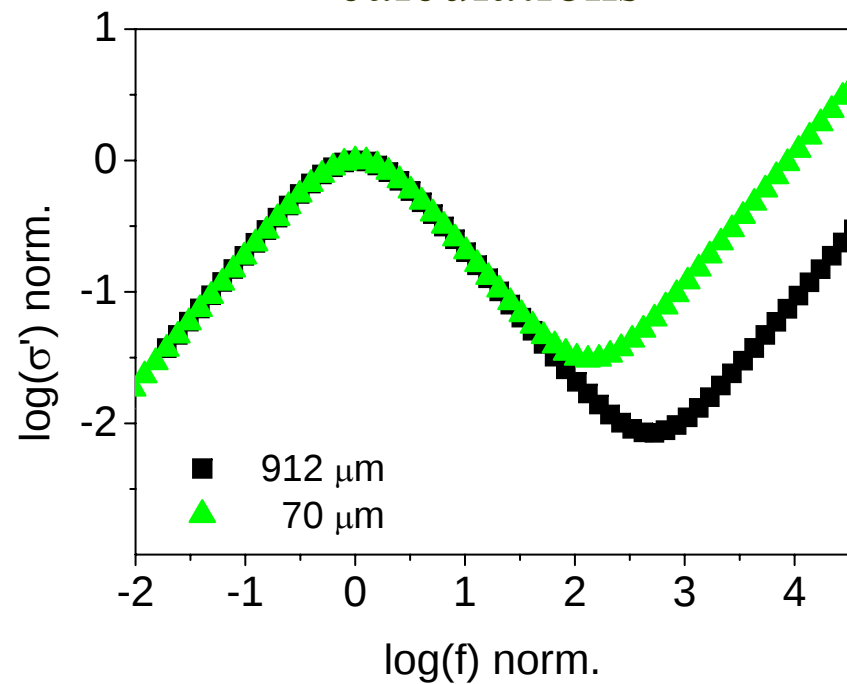


Dielectric spectra of ionic liquids - length dependence, scaling -

experiment



calculations



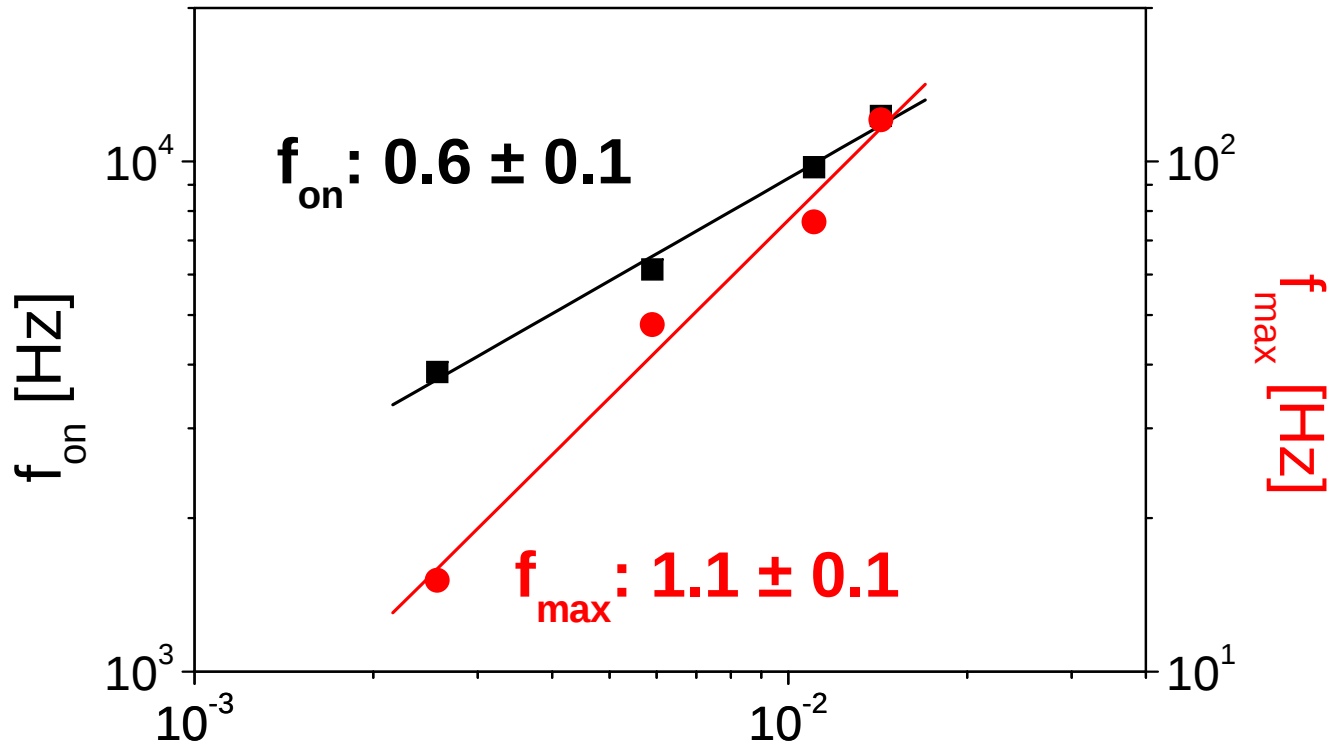
$$\frac{d}{d\omega}(\sigma''_{\text{meas}}) = 0 \Rightarrow f_{\text{on}} \cong \frac{\sigma_0}{\varepsilon_0} \frac{1}{\sqrt{\varepsilon'_b \varepsilon'_i}} \sqrt{\frac{2d_i}{L}} \text{ and } f_{\text{max}} \cong \frac{\sigma_0}{\varepsilon_0} \frac{1}{2\pi\varepsilon'_i} \frac{2d_i}{L}$$

$$f_{\text{on}} \sim \frac{1}{\sqrt{L}}, \text{ while } f_{\text{max}} \sim \frac{1}{L}$$

Dielectric spectra of ionic liquids

- length dependence, scaling -

experiment



$$f_{\text{on}} \sim \frac{1}{\sqrt{L}} \Rightarrow \text{slope of } 0.5$$

$$f_{\text{max}} \sim \frac{1}{L} \Rightarrow \text{slope of } 1.0$$

Summary for EP

1. What is the *signature of electrode polarisation (EP)* and how can it be analysed *quantitatively*?

EP has a peculiar signature in the complex dielectric function and conductivity but as well the length of the sample cell.

2. What *quantitative information* can be deduced *from fits of the EP*?

With a novel formula *discovered by A. Serghei* σ_0 can be deduced from $f_{\min}$ and $f_{\max}$ of $\sigma''(f)$.

3. What is the *quantitative model* to describe EP?

A model adding the *complex impedances* at the interface and in the bulk.

4. What *information can be deduced* from EP?

A detailed understanding of the *geometry and the complex dielectric function* of the interphase is in reach.

Thanks to



...DFG for financial support and to you for attention